

A Regret Theory and Frank Aggregation Operator-Based Spherical Cubic Fuzzy CoCoSo Decision Method

Yuan Rong^{1,*}, Huiyi Qiu¹, Qian Zhang¹, Wenxue Zhang¹

¹ School of medical information and engineering, Ningxia Medical University, 750004, Yinchuan, China

ARTICLE INFO

Article history:

Received 22 July 2026

Received in revised form 8 September 2026

Accepted 6 October 2026

Available online 10 October 2026

Keywords:

Spherical cubic fuzzy set; Frank t-norm and s-norm; LOPCOW; CoCoSo

ABSTRACT

As an extension of spherical fuzzy sets, spherical cubic fuzzy (SCF) sets can more effectively handle uncertain data represented by cubic fuzzy numbers. Nevertheless, existing research on SCF sets is primarily focused on algebraic operations and rarely considers the psychological behavior of experts in decision-making. To address these limitations, this study proposes a novel decision-making method based on Frank aggregation operators and a regret theory-based Combined Compromise Solution (CoCoSo) model in a spherical cubic fuzzy environment. Specifically, SCF sets are used to represent uncertain preference information by combining the advantages of spherical fuzzy sets and cubic sets. We then define new SCF operations based on Frank norms and propose four SCF aggregation operators to obtain collective assessments from multiple experts. Subsequently, the criteria weights are determined using an enhanced logarithmic percentage change-driven objective weighting (LOPCOW) method incorporating the SCF score function. To rank the alternatives while considering the psychological behavior of decision-makers, we develop an improved CoCoSo model by integrating regret theory with the proposed SCF aggregation operators. The proposed decision-making method is applied to the diagnosis of patients with cardiovascular disease under uncertainty. Sensitivity and comparative analyses are conducted to evaluate the robustness and effectiveness of the proposed regret theory-based spherical cubic fuzzy CoCoSo decision-making method.

1. Introduction

Decision making is the process in which decision experts provide reliable evaluation information for alternative solutions based on multiple criteria involved in the problem and their own cognitive abilities and knowledge background and then determine the optimal choice or recommended solution. As an important branch of modern decision science, multiple criteria decision-making (MCDM) has become a widely used supporting tool in the evaluation process, scheme selection and intelligent recommendation. Among them, preference information, criteria and decision methods are the research hotspots in the MCDM analysis process. However, the real decision environment is highly complex, ambiguous and uncertain. Due to the ambiguity of human cognition and the

* Corresponding author.

E-mail address: rongyuanry@163.com

incompleteness of information, decision experts often find it difficult to provide accurate evaluation information, which makes it difficult to directly conduct decision analysis. Therefore, MCDM methods based on uncertain information are increasingly receiving attention from scholars. How to express decisionmakers' uncertain preference information more accurately and comprehensively and how to obtain evaluation information that is more in line with decisionmakers' behavioral expression habits has become an important research direction in uncertain decision analysis.

To effectively characterize fuzzy and uncertain information, Zadeh [1] creatively originated using membership functions to characterize fuzzy information and then established fuzzy sets (FSs) theory, which has been widely applied in practical decision analysis. To further compensate for the shortcomings of FSs in representing uncertain information, Atanassov [2] developed intuitionistic FSs based on the consideration of non-membership and hesitation degrees, thereby enhancing its ability for representing uncertain information. Subsequently, a series of new FSs theories were presented, such as Pythagorean FSs [3], q-rung orthopair FSs [4], picture FSs [5] and so on. Since its proposal, the FSs theory mentioned above has received widespread attention from scholars and has achieved fruitful research results in various fields [6–8]. As an effective extension of picture FSs, spherical FSs have attracted much attention because they offer decision experts with greater flexibility to represent uncertain assessment information. The sum of squares of membership, neutrality and non-membership degrees needs to be within the interval $[0, 1]$. The proposal of spherical FSs has laid a solid theoretical foundation for the analysis of complex decision problems, and scholars have conducted extensive research on its applications in information fusion, information measures, decision methods and applications. Although the above FSs theory and its extensions demonstrate strong information representation capabilities, they mainly represent membership and non-membership information with precise numbers, which makes it difficult to characterize the inherent fuzziness of preference information in complex decision contexts. In view of this, Ayaz et al., [9] combines spherical FSs with cubic sets to propose spherical cubic fuzzy (SCF) sets. SCF sets are a combination of spherical FSs and cubic sets, and its membership, neutrality and non-membership grades are all expressed in the form of cubic fuzzy numbers. Compared to interval-valued spherical fuzzy numbers, SCF sets can more accurately characterize uncertainty, which makes them an important tool for experts to handle complex uncertain decision problems. For instance, Ayaz et al., [9] presented some novel Hamacher aggregation operators and constructed decision method for evaluating enterprise production. Ali et al., [10] presented a generalized ELiminating Et Choice Translating REality (ELECTRE) method with SCF sets to construct group decision framework. Mohammed *et al.*, [11] introduced a hybrid SCF group decision framework based on fuzzy decision by opinion score method (FDOSM), fuzzy weighted with zero inconsistency criterion (FWZIC) method and Preference Ranking Organization Method for Enrichment of Evaluations (PROMETHEE) method to prioritize reactive fault tolerance categories. However, the existing research shows that these introduced methods fail to consider the psychological behavior of experts in decision analysis process.

In the process of MCDM analysis, the decision method is a key element in determining the optimal ranking and therefore receives much attention from scholars. At present, a series of decision methods have been proposed and applied to practical decision problems, such as TOPSIS method [12], WASPAS method [13] and ARAS method [14], PROMETHEE method [15] and so forth. Besides, Yazdani *et al.*, [16] proposed the Combined Compromise Solution (CoCoSo) based on weighted sum model, weight product model and three novel fusion strategies. This method first calculates the comprehensive evaluation value of alternative solutions through weighted sum and weight product models and then uses multiple aggregation strategies to determine the final utility value of alternative solutions, which provides a more balanced, stable and interpretable ranking result for complex decision problems. Compared to traditional decision methods, CoCoSo method performs

better in reducing aggregation bias, avoiding ranking reversal and has better applicability in practical decision problems. Given the advantages of CoCoSo method in decision analysis, this method has been extended to different uncertain environments for handling practical decision-making problems [17–20]. For instance, Rong and Yu [21] presented an integrated CoCoSo method based on generalized Dombi operators to the prioritization of offshore wind farm site. Yu *et al.*, [22] propounded an improved FBWM-CoCoSo method to assess the green governance capabilities of cities on China. Khan *et al.*, [23] developed the extended (p,q) -rung orthopair fuzzy CoCoSo method based on Sugeno-Weber aggregation operator for the management of health care waste. However, the CoCoSo extended model mentioned above rarely considers the psychological behavior of decisionmakers. Zhang and Wei [24] constructed the spherical fuzzy CoCoSo method based on cumulative theory to select the optimal electric vehicles charging stations. Fan *et al.*, [25] presented a novel probabilistic uncertain linguistic failure mode and effect analysis method based on regret theory and CoCoSo to assess the failure mode of basin-type insulators. Up to now, there is no research indicating that CoCoSo method and regret theory have been extended to SCF environment for constructing decision approach.

Based on the prior research works, the research challenges of this study are identified as follows:

- i. Prior SCF operations are mostly defined based on algebraic norms. The flexibility of the operations such as Frank norms have not been fully considered, which limits the flexibility of SCF information fusion.
- ii. The traditional LOPCOW method has not been effectively applied to the SCF environment and criterion weight determination in SCF decision still suffers from strong subjectivity or insufficient objectivity.
- iii. The classical CoCoSo model has not been systematically integrated with regret theory and SCF Frank aggregation operators, making it difficult to simultaneously address SCF decision problem with uncertainty and psychological behavior.

Considering the challenges and motivations mentioned, the novelties of this study are analyzed as below:

- i. In response to the limitation of existing SCF set operations to algebraic t-nor operations, this paper introduces new SCF Frank operations based on Frank norms. And four new types of SCF Frank aggregation operators are presented for fusing multiple SCF numbers. This work enriches the ensemble operators in SCF environment and provides a more flexible fusion tool for handling uncertain and cubic fuzzy information.
- ii. This article extends the classical LOPCOW method using the SCF score function to determine the criterion weight. This method can reduce the bias caused by subjective weighting, making the acquisition of criterion weights more objective and reasonable.
- iii. To address the issue that traditional SCF decision methods rarely consider experts' psychological behavior, this paper combines regret theory with the CoCoSo model to construct an improved SCF-RT-CoCoSo decision methodology. This methodology can not only tackle uncertain information with the aids of SCF sets but also reflect experts' psychological behavior in uncertain environments, which makes the decision results closer to actual decision contexts.
- iv. This study exploits the introduced SCF-LOPCOW-RT-CoCoSo decision methodology to the diagnosis process of patients with cardiovascular disease under an uncertain setting and verifies the robustness and effectiveness of the methodology by sensitivity analysis and comparative discussions. The results indicate that the developed method can effectively handle complex decision problems.

The ensuing sections of this paper are structured as follows. Section 2 revisits the foundational conceptions of cubic set, SCF sets and regret theory. Section 3 formulates operational rules for SCF numbers grounded Frank norms and propose several novel SCF Frank aggregation operators. Section 4 expounds on the SCF-LOPCOW-RT-CoCoSo decision framework and delineates its detailed decision procedures. Section 5 showcases a case study concerning the diagnosis process of cardiovascular disease patients, accompanied by sensitivity analysis and comparative discussions. Ultimately, Section 6 brings the study to a close with concluding remarks.

2. Preliminaries

This part gives several basic conceptions of this manuscript.

Definition 1 [26]. A cubic set Γ on Y is defined as below:

$$\Gamma = \left\{ \langle y, A_\Gamma(y), B_\Gamma(y) \rangle \mid y \in Y \right\} \quad (1)$$

where $A_\Gamma : Y \rightarrow [0,1]$ and $B_\Gamma : Y \rightarrow [0,1]$ denote the membership and non-membership grades, respectively.

Definition 2 [27]. A spherical fuzzy set Θ on Y is defined as below:

$$\Theta = \left\{ \langle y, \zeta_\Theta(y), \psi_\Theta(y), \xi_\Theta(y) \rangle \mid y \in Y \right\} \quad (2)$$

wherein $\zeta_\Theta(y), \psi_\Theta(y), \xi_\Theta(y) : Y \rightarrow [0,1]$ indicate the positive, neutral, and negative membership function of an objective $y \in Y$ to Θ , respectively. The triple $(\zeta_\Theta(y), \psi_\Theta(y), \xi_\Theta(y))$ meets the condition $0 \leq (\zeta_\Theta(y))^2 + (\psi_\Theta(y))^2 + (\xi_\Theta(y))^2 \leq 1$.

Definition 3 [9]. A SCF set Ψ on Y is articulated as below:

$$\Psi = \left\{ \langle y, F_\Psi(y), G_\Psi(y), H_\Psi(y) \rangle \mid y \in Y \right\} \quad (3)$$

wherein $F_\Psi(y) = ([f_\Psi^-, f_\Psi^+], f_\Psi)$, $G_\Psi(y) = ([g_\Psi^-, g_\Psi^+], g_\Psi)$ and $H_\Psi(y) = ([h_\Psi^-, h_\Psi^+], h_\Psi)$ indicate the positive, neutral, and negative membership grades of an objective $y \in Y$ to Ψ respectively, which satisfies the conditions that $0 \leq \left(\sup([f_\Psi^-, f_\Psi^+]) \right)^2 + \left(\sup([g_\Psi^-, g_\Psi^+]) \right)^2 + \left(\sup([h_\Psi^-, h_\Psi^+]) \right)^2 \leq 1$ and $0 \leq (f_\Psi)^2 + (g_\Psi)^2 + (h_\Psi)^2 \leq 1$. The hesitant grade of SCF set is defined as

$$\pi_\Psi(y) = \left(\sqrt{1 - \left(\left(\sup([f_\Psi^-, f_\Psi^+]) \right)^2 + \left(\sup([g_\Psi^-, g_\Psi^+]) \right)^2 + \left(\sup([h_\Psi^-, h_\Psi^+]) \right)^2 \right)}, \sqrt{1 - ((f_\Psi)^2 + (g_\Psi)^2 + (h_\Psi)^2)} \right). \quad \text{For}$$

convenience, a SCF number (SCFN) is depicted as:

$$\Psi = (F_\Psi(y), G_\Psi(y), H_\Psi(y)) = \left\{ ([f_\Psi^-, f_\Psi^+], f_\Psi), ([g_\Psi^-, g_\Psi^+], g_\Psi), ([h_\Psi^-, h_\Psi^+], h_\Psi) \right\}.$$

Definition 4 [9]. Let $\Psi = \left\{ ([f_\Psi^-, f_\Psi^+], f_\Psi), ([g_\Psi^-, g_\Psi^+], g_\Psi), ([h_\Psi^-, h_\Psi^+], h_\Psi) \right\}$

$\Psi_s = \left\{ ([f_{\Psi_s}^-, f_{\Psi_s}^+], f_{\Psi_s}), ([g_{\Psi_s}^-, g_{\Psi_s}^+], g_{\Psi_s}), ([h_{\Psi_s}^-, h_{\Psi_s}^+], h_{\Psi_s}) \right\} (s=1,2)$ be three SCFNs and a real number λ , the operations of SCFNs are depicted as:

$$\begin{aligned}
 (1) \Psi_1 \oplus \Psi_2 &= \left\{ \left(\left[\begin{array}{c} \sqrt{(f_{\Psi_1}^-)^2 + (f_{\Psi_2}^-)^2 - (f_{\Psi_1}^-)^2 (f_{\Psi_2}^-)^2} \\ \sqrt{(f_{\Psi_1}^-)^2 + (f_{\Psi_2}^+)^2 - (f_{\Psi_1}^+)^2 (f_{\Psi_2}^+)^2} \\ \sqrt{(f_{\Psi_1}^+)^2 + (f_{\Psi_2}^+)^2 - (f_{\Psi_1}^+)^2 (f_{\Psi_2}^+)^2} \end{array} \right], \left[\begin{array}{c} g_{\Psi_1}^- g_{\Psi_2}^-, \\ g_{\Psi_1}^+ g_{\Psi_2}^+, \\ g_{\Psi_1} g_{\Psi_2} \end{array} \right], \left[\begin{array}{c} h_{\Psi_1}^- h_{\Psi_2}^-, \\ h_{\Psi_1}^+ h_{\Psi_2}^+, \\ h_{\Psi_1} h_{\Psi_2} \end{array} \right] \right\}; \\
 (2) \Psi_1 \otimes \Psi_2 &= \left\{ \left(\left[\begin{array}{c} f_{\Psi_1}^- f_{\Psi_2}^-, \\ f_{\Psi_1}^+ f_{\Psi_2}^+, \\ f_{\Psi_1} f_{\Psi_2} \end{array} \right], \left[\begin{array}{c} g_{\Psi_1}^- g_{\Psi_2}^-, \\ g_{\Psi_1}^+ g_{\Psi_2}^+, \\ g_{\Psi_1} g_{\Psi_2} \end{array} \right], \left[\begin{array}{c} \sqrt{(h_{\Psi_1}^-)^2 + (h_{\Psi_2}^-)^2 - (h_{\Psi_1}^-)^2 (h_{\Psi_2}^-)^2} \\ \sqrt{(h_{\Psi_1}^-)^2 + (h_{\Psi_2}^+)^2 - (h_{\Psi_1}^+)^2 (h_{\Psi_2}^+)^2} \\ \sqrt{(h_{\Psi_1}^+)^2 + (h_{\Psi_2}^+)^2 - (h_{\Psi_1}^+)^2 (h_{\Psi_2}^+)^2} \end{array} \right] \right\}; \\
 (3) \lambda \Psi &= \left\{ \left(\left[\begin{array}{c} \sqrt{1 - (1 - (f_{\Psi}^-)^2)^\lambda} \\ \sqrt{1 - (1 - (f_{\Psi}^+)^2)^\lambda} \\ \sqrt{1 - (1 - (f_{\Psi})^2)^\lambda} \end{array} \right], \left[\begin{array}{c} (g_{\Psi}^-)^\lambda, (g_{\Psi}^+)^\lambda \\ (g_{\Psi})^\lambda \end{array} \right], \left[\begin{array}{c} (h_{\Psi}^-)^\lambda, (h_{\Psi}^+)^\lambda \\ (h_{\Psi})^\lambda \end{array} \right] \right\}; \\
 (4) (\Psi)^\lambda &= \left\{ \left(\left[\begin{array}{c} (f_{\Psi}^-)^\lambda, (f_{\Psi}^+)^\lambda \\ (f_{\Psi})^\lambda \end{array} \right], \left[\begin{array}{c} (g_{\Psi}^-)^\lambda, (g_{\Psi}^+)^\lambda \\ (g_{\Psi})^\lambda \end{array} \right], \left[\begin{array}{c} \sqrt{1 - (1 - (h_{\Psi}^-)^2)^\lambda} \\ \sqrt{1 - (1 - (h_{\Psi}^+)^2)^\lambda} \\ \sqrt{1 - (1 - (h_{\Psi})^2)^\lambda} \end{array} \right] \right\}.
 \end{aligned}$$

Definition 5 [9]. Considering a SCFN $\Psi = \{([f_{\Psi}^-, f_{\Psi}^+], f_{\Psi}), ([g_{\Psi}^-, g_{\Psi}^+], g_{\Psi}), ([h_{\Psi}^-, h_{\Psi}^+], h_{\Psi})\}$, then the score function $SF(\Psi)$ of Ψ is defined as:

$$SF(\Psi) = \frac{1}{2} \left(1 + \frac{1}{9} \left((f_{\Psi}^- + f_{\Psi}^+ + f_{\Psi})^2 + (g_{\Psi}^- + g_{\Psi}^+ + g_{\Psi})^2 - (h_{\Psi}^- + h_{\Psi}^+ + h_{\Psi})^2 \right) \right), \quad (4)$$

Where $SF(\Psi) \in [0, 1]$.

Definition 6 [9]. For a SCFN $\Psi = \{([f_{\Psi}^-, f_{\Psi}^+], f_{\Psi}), ([g_{\Psi}^-, g_{\Psi}^+], g_{\Psi}), ([h_{\Psi}^-, h_{\Psi}^+], h_{\Psi})\}$, then the accuracy function $AF(\Psi)$ of Ψ is defined as:

$$AF(\Psi) = \frac{1}{9} \left((f_{\Psi}^- + f_{\Psi}^+ + f_{\Psi})^2 + (g_{\Psi}^- + g_{\Psi}^+ + g_{\Psi})^2 + (h_{\Psi}^- + h_{\Psi}^+ + h_{\Psi})^2 \right). \quad (5)$$

Where $AF(\Psi) \in [0, 1]$

Definition 7 [9]. Suppose that $\Psi_s = \{([f_{\Psi_s}^-, f_{\Psi_s}^+], f_{\Psi_s}), ([g_{\Psi_s}^-, g_{\Psi_s}^+], g_{\Psi_s}), ([h_{\Psi_s}^-, h_{\Psi_s}^+], h_{\Psi_s})\} (s=1, 2)$ are two SCFNs, the comparison rules are given as:

If $SF(\Psi_1) > SF(\Psi_2)$, then $\Psi_1 \succ \Psi_2$;

If $SF(\Psi_1) < SF(\Psi_2)$, then $\Psi_1 \prec \Psi_2$;

If $SF(\Psi_1) = SF(\Psi_2)$, then

If $AF(\Psi_1) > AF(\Psi_2)$, then $\Psi_1 \succ \Psi_2$;

If $AF(\Psi_1) < AF(\Psi_2)$, then $\Psi_1 \prec \Psi_2$;

If $AF(\Psi_1) = AF(\Psi_2)$, then $\Psi_1 \sim \Psi_2$.

Definition 8 [9]. It is supposed that $\Psi_s = \{([f_{\Psi_s}^-, f_{\Psi_s}^+], f_{\Psi_s}), ([g_{\Psi_s}^-, g_{\Psi_s}^+], g_{\Psi_s}), ([h_{\Psi_s}^-, h_{\Psi_s}^+], h_{\Psi_s})\} (s=1,2)$ be a set of SCFNs. Λ_s is the weight of SCFN Ψ_s with $\sum_{s=1}^n \Lambda_s = 1, \Lambda_s \in [0, 1]$. Then SCF weighted averaging operator (SCFWA) operator is depicted as:

$$SCFWA(\Psi_1, \Psi_2, \dots, \Psi_n) = \bigoplus_{s=1}^n \Lambda_s \Psi_s$$

$$= \left\{ \left(\left[\sqrt{1 - \prod_{s=1}^n \left(1 - (f_{\Psi_s}^-)^2\right)^{\Lambda_s}} \right], \left[\sqrt{1 - \prod_{s=1}^n \left(1 - (f_{\Psi_s}^+)^2\right)^{\Lambda_s}} \right], \sqrt{1 - \prod_{s=1}^n \left(1 - (f_{\Psi_s})^2\right)^{\Lambda_s}} \right), \left(\left[\prod_{s=1}^n (g_{\Psi_s}^-)^{\Lambda_s} \right], \left[\prod_{s=1}^n (g_{\Psi_s}^+)^{\Lambda_s} \right], \prod_{s=1}^n (g_{\Psi_s})^{\Lambda_s} \right), \left(\left[\prod_{s=1}^n (h_{\Psi_s}^-)^{\Lambda_s} \right], \left[\prod_{s=1}^n (h_{\Psi_s}^+)^{\Lambda_s} \right], \prod_{s=1}^n (h_{\Psi_s})^{\Lambda_s} \right) \right\} \quad (6)$$

Definition 9 [9]. It is supposed that $\Psi_s = \{([f_{\Psi_s}^-, f_{\Psi_s}^+], f_{\Psi_s}), ([g_{\Psi_s}^-, g_{\Psi_s}^+], g_{\Psi_s}), ([h_{\Psi_s}^-, h_{\Psi_s}^+], h_{\Psi_s})\} (s=1,2)$ be a set of SCFNs. Λ_s is the weight of SCFN Ψ_s with $\sum_{s=1}^n \Lambda_s = 1, \Lambda_s \in [0, 1]$. Then SCF weighted geometric operator (SCFWG) operator is depicted as:

$$SCFWG(\Psi_1, \Psi_2, \dots, \Psi_n) = \bigotimes_{s=1}^n (\Psi_s)^{\Lambda_s}$$

$$= \left\{ \left(\left[\prod_{s=1}^n (f_{\Psi_s}^-)^{\Lambda_s} \right], \left[\prod_{s=1}^n (f_{\Psi_s}^+)^{\Lambda_s} \right], \prod_{s=1}^n (f_{\Psi_s})^{\Lambda_s} \right), \left(\left[\prod_{s=1}^n (g_{\Psi_s}^-)^{\Lambda_s} \right], \left[\prod_{s=1}^n (g_{\Psi_s}^+)^{\Lambda_s} \right], \prod_{s=1}^n (g_{\Psi_s})^{\Lambda_s} \right), \left(\left[\sqrt{1 - \prod_{s=1}^n \left(1 - (h_{\Psi_s}^-)^2\right)^{\Lambda_s}} \right], \left[\sqrt{1 - \prod_{s=1}^n \left(1 - (h_{\Psi_s}^+)^2\right)^{\Lambda_s}} \right], \sqrt{1 - \prod_{s=1}^n \left(1 - (h_{\Psi_s})^2\right)^{\Lambda_s}} \right) \right\}. \quad (7)$$

Definition 10 [28]. Let $a, \hat{b} \in [0, 1]$ be two real numbers. Then the Frank t-norm and s-norm are stated as

$$T_F(a, \hat{b}) = a \otimes_F \hat{b} = \log_Y \left(1 + \frac{(\Upsilon^a - 1)(\Upsilon^{\hat{b}} - 1)}{\Upsilon - 1} \right) \quad (8)$$

$$S_F(a, \hat{b}) = a \oplus_F \hat{b} = 1 - \log_Y \left(1 + \frac{(\Upsilon^{1-a} - 1)(\Upsilon^{1-\hat{b}} - 1)}{\Upsilon - 1} \right) \quad (9)$$

where $\Upsilon \in (1, +\infty)$. It is obvious that Frank t-norm and Frank s-norm possess two special cases:

- (1) If $\Upsilon \rightarrow 1$, the Frank t-norm and Frank s-norms are degenerated into the Algebraic operations.
- (2) If $\Upsilon \rightarrow \infty$, the Frank product and Frank sum are reduced to the Lukasiewicz operations.

3. Some novel SCF Frank operators

This section first defines the Frank operations on SCFNs and then proposes four types Frank aggregation operators.

3.1 Frank operations of SCFNs

Definition 11. Let $\Psi = \left\{ \left(\left[f_{\Psi}^{-}, f_{\Psi}^{+} \right], f_{\Psi} \right), \left(\left[g_{\Psi}^{-}, g_{\Psi}^{+} \right], g_{\Psi} \right), \left(\left[h_{\Psi}^{-}, h_{\Psi}^{+} \right], h_{\Psi} \right) \right\}$ and

$\Psi_s = \left\{ \left(\left[f_{\Psi_s}^{-}, f_{\Psi_s}^{+} \right], f_{\Psi_s} \right), \left(\left[g_{\Psi_s}^{-}, g_{\Psi_s}^{+} \right], g_{\Psi_s} \right), \left(\left[h_{\Psi_s}^{-}, h_{\Psi_s}^{+} \right], h_{\Psi_s} \right) \right\} (s=1,2)$ be three SCFNs and a real number

λ , then the SCF Frank operations are defined as:

$$\Psi_1 \oplus_F \Psi_2 = \left\{ \left[\left(\sqrt[1 - \log_{\Upsilon} \left(1 + \frac{\left(\Upsilon^{1 - (f_{\Psi_1}^{-})^2} - 1 \right) \left(\Upsilon^{1 - (f_{\Psi_2}^{-})^2} - 1 \right)}{\Upsilon - 1} \right)} \right), \sqrt[1 - \log_{\Upsilon} \left(1 + \frac{\left(\Upsilon^{1 - (f_{\Psi_1}^{+})^2} - 1 \right) \left(\Upsilon^{1 - (f_{\Psi_2}^{+})^2} - 1 \right)}{\Upsilon - 1} \right)} \right], \sqrt[1 - \log_{\Upsilon} \left(1 + \frac{\left(\Upsilon^{1 - (f_{\Psi_1})^2} - 1 \right) \left(\Upsilon^{1 - (f_{\Psi_2})^2} - 1 \right)}{\Upsilon - 1} \right)} \right], \left[\left(\sqrt[\log_{\Upsilon} \left(1 + \frac{\left(\Upsilon^{(g_{\Psi_1}^{-})^2} - 1 \right) \left(\Upsilon^{(g_{\Psi_2}^{-})^2} - 1 \right)}{\Upsilon - 1} \right)} \right), \sqrt[\log_{\Upsilon} \left(1 + \frac{\left(\Upsilon^{(g_{\Psi_1}^{+})^2} - 1 \right) \left(\Upsilon^{(g_{\Psi_2}^{+})^2} - 1 \right)}{\Upsilon - 1} \right)} \right], \sqrt[\log_{\Upsilon} \left(1 + \frac{\left(\Upsilon^{(g_{\Psi_1})^2} - 1 \right) \left(\Upsilon^{(g_{\Psi_2})^2} - 1 \right)}{\Upsilon - 1} \right)} \right], \left[\left(\sqrt[\log_{\Upsilon} \left(1 + \frac{\left(\Upsilon^{(h_{\Psi_1}^{-})^2} - 1 \right) \left(\Upsilon^{(h_{\Psi_2}^{-})^2} - 1 \right)}{\Upsilon - 1} \right)} \right), \sqrt[\log_{\Upsilon} \left(1 + \frac{\left(\Upsilon^{(h_{\Psi_1}^{+})^2} - 1 \right) \left(\Upsilon^{(h_{\Psi_2}^{+})^2} - 1 \right)}{\Upsilon - 1} \right)} \right], \sqrt[\log_{\Upsilon} \left(1 + \frac{\left(\Upsilon^{(h_{\Psi_1})^2} - 1 \right) \left(\Upsilon^{(h_{\Psi_2})^2} - 1 \right)}{\Upsilon - 1} \right)} \right] \right\};$$

$$\begin{aligned}
 \Psi_1 \otimes_F \Psi_2 = & \left\{ \left[\sqrt{\log_\Upsilon \left(1 + \frac{\left(\Upsilon^{(f_{\Psi_1}^-)^2} - 1 \right) \left(\Upsilon^{(f_{\Psi_2}^-)^2} - 1 \right)}{\Upsilon - 1} \right)}, \sqrt{\log_\Upsilon \left(1 + \frac{\left(\Upsilon^{(g_{\Psi_1}^-)^2} - 1 \right) \left(\Upsilon^{(g_{\Psi_2}^-)^2} - 1 \right)}{\Upsilon - 1} \right)}, \right. \right. \\
 & \left[\sqrt{\log_\Upsilon \left(1 + \frac{\left(\Upsilon^{(f_{\Psi_1}^+)^2} - 1 \right) \left(\Upsilon^{(f_{\Psi_2}^+)^2} - 1 \right)}{\Upsilon - 1} \right)}, \sqrt{\log_\Upsilon \left(1 + \frac{\left(\Upsilon^{(g_{\Psi_1}^+)^2} - 1 \right) \left(\Upsilon^{(g_{\Psi_2}^+)^2} - 1 \right)}{\Upsilon - 1} \right)}, \right. \\
 & \left. \left. \sqrt{\log_\Upsilon \left(1 + \frac{\left(\Upsilon^{(f_{\Psi_1})^2} - 1 \right) \left(\Upsilon^{(f_{\Psi_2})^2} - 1 \right)}{\Upsilon - 1} \right)}, \sqrt{\log_\Upsilon \left(1 + \frac{\left(\Upsilon^{(g_{\Psi_1})^2} - 1 \right) \left(\Upsilon^{(g_{\Psi_2})^2} - 1 \right)}{\Upsilon - 1} \right)} \right] \right\}; \\
 & \left\{ \left[\sqrt{1 - \log_\Upsilon \left(1 + \frac{\left(\Upsilon^{1 - (h_{\Psi_1}^-)^2} - 1 \right) \left(\Upsilon^{1 - (h_{\Psi_2}^-)^2} - 1 \right)}{\theta - 1} \right)}, \sqrt{1 - \log_\Upsilon \left(1 + \frac{\left(\Upsilon^{1 - (h_{\Psi_1}^+)^2} - 1 \right) \left(\Upsilon^{1 - (h_{\Psi_2}^+)^2} - 1 \right)}{\theta - 1} \right)}, \right. \right. \\
 & \left. \left. \sqrt{1 - \log_\Upsilon \left(1 + \frac{\left(\Upsilon^{1 - (h_{\Psi_1})^2} - 1 \right) \left(\Upsilon^{1 - (h_{\Psi_2})^2} - 1 \right)}{\theta - 1} \right)} \right] \right\}; \\
 \lambda^\bullet_F \Psi = & \left\{ \left[\sqrt{1 - \log_\Upsilon \left(1 + \frac{\left(\Upsilon^{1 - (f_{\Psi}^-)^2} - 1 \right)^\lambda}{(\Upsilon - 1)^{\lambda - 1}} \right)}, \sqrt{\log_\theta \left(1 + \frac{\left(\Upsilon^{(g_{\Psi}^-)^2} - 1 \right)^\lambda}{(\Upsilon - 1)^{\lambda - 1}} \right)}, \right. \right. \\
 & \left[\sqrt{1 - \log_\Upsilon \left(1 + \frac{\left(\Upsilon^{1 - (f_{\Psi}^+)^2} - 1 \right)^\lambda}{(\Upsilon - 1)^{\lambda - 1}} \right)}, \sqrt{\log_\theta \left(1 + \frac{\left(\Upsilon^{(g_{\Psi}^+)^2} - 1 \right)^\lambda}{(\Upsilon - 1)^{\lambda - 1}} \right)}, \right. \\
 & \left. \left. \sqrt{1 - \log_\Upsilon \left(1 + \frac{\left(\Upsilon^{1 - (f_{\Psi})^2} - 1 \right)^\lambda}{(\Upsilon - 1)^{\lambda - 1}} \right)}, \sqrt{\log_\theta \left(1 + \frac{\left(\Upsilon^{(g_{\Psi})^2} - 1 \right)^\lambda}{(\Upsilon - 1)^{\lambda - 1}} \right)} \right] \right\};
 \end{aligned}$$

$$(\Psi)^{\wedge_F \lambda} = \left\{ \left[\sqrt{\log_{\theta} \left(1 + \frac{\left(\Upsilon^{(f_{\Psi}^-)^2} - 1 \right)^{\lambda}}{(\Upsilon - 1)^{\lambda-1}} \right)}, \sqrt{\log_{\theta} \left(1 + \frac{\left(\Upsilon^{(f_{\Psi}^+)^2} - 1 \right)^{\lambda}}{(\Upsilon - 1)^{\lambda-1}} \right)}, \sqrt{\log_{\theta} \left(1 + \frac{\left(\Upsilon^{(g_{\Psi}^-)^2} - 1 \right)^{\lambda}}{(\Upsilon - 1)^{\lambda-1}} \right)}, \sqrt{\log_{\theta} \left(1 + \frac{\left(\Upsilon^{(g_{\Psi}^+)^2} - 1 \right)^{\lambda}}{(\Upsilon - 1)^{\lambda-1}} \right)}, \sqrt{1 - \log_{\Upsilon} \left(1 + \frac{\left(\Upsilon^{1-(h_{\Psi}^-)^2} - 1 \right)^{\lambda}}{(\Upsilon - 1)^{\lambda-1}} \right)}, \sqrt{1 - \log_{\Upsilon} \left(1 + \frac{\left(\Upsilon^{1-(h_{\Psi}^+)^2} - 1 \right)^{\lambda}}{(\Upsilon - 1)^{\lambda-1}} \right)} \right], \left[\sqrt{\log_{\theta} \left(1 + \frac{\left(\Upsilon^{(f_{\Psi}^-)^2} - 1 \right)^{\lambda}}{(\Upsilon - 1)^{\lambda-1}} \right)}, \sqrt{\log_{\theta} \left(1 + \frac{\left(\Upsilon^{(f_{\Psi}^+)^2} - 1 \right)^{\lambda}}{(\Upsilon - 1)^{\lambda-1}} \right)}, \sqrt{\log_{\theta} \left(1 + \frac{\left(\Upsilon^{(g_{\Psi}^-)^2} - 1 \right)^{\lambda}}{(\Upsilon - 1)^{\lambda-1}} \right)}, \sqrt{\log_{\theta} \left(1 + \frac{\left(\Upsilon^{(g_{\Psi}^+)^2} - 1 \right)^{\lambda}}{(\Upsilon - 1)^{\lambda-1}} \right)}, \sqrt{1 - \log_{\Upsilon} \left(1 + \frac{\left(\Upsilon^{1-(h_{\Psi}^-)^2} - 1 \right)^{\lambda}}{(\Upsilon - 1)^{\lambda-1}} \right)}, \sqrt{1 - \log_{\Upsilon} \left(1 + \frac{\left(\Upsilon^{1-(h_{\Psi}^+)^2} - 1 \right)^{\lambda}}{(\Upsilon - 1)^{\lambda-1}} \right)} \right], \left[\sqrt{\log_{\theta} \left(1 + \frac{\left(\Upsilon^{(f_{\Psi}^-)^2} - 1 \right)^{\lambda}}{(\Upsilon - 1)^{\lambda-1}} \right)}, \sqrt{\log_{\theta} \left(1 + \frac{\left(\Upsilon^{(f_{\Psi}^+)^2} - 1 \right)^{\lambda}}{(\Upsilon - 1)^{\lambda-1}} \right)}, \sqrt{\log_{\theta} \left(1 + \frac{\left(\Upsilon^{(g_{\Psi}^-)^2} - 1 \right)^{\lambda}}{(\Upsilon - 1)^{\lambda-1}} \right)}, \sqrt{\log_{\theta} \left(1 + \frac{\left(\Upsilon^{(g_{\Psi}^+)^2} - 1 \right)^{\lambda}}{(\Upsilon - 1)^{\lambda-1}} \right)}, \sqrt{1 - \log_{\Upsilon} \left(1 + \frac{\left(\Upsilon^{1-(h_{\Psi}^-)^2} - 1 \right)^{\lambda}}{(\Upsilon - 1)^{\lambda-1}} \right)}, \sqrt{1 - \log_{\Upsilon} \left(1 + \frac{\left(\Upsilon^{1-(h_{\Psi}^+)^2} - 1 \right)^{\lambda}}{(\Upsilon - 1)^{\lambda-1}} \right)} \right] \right\}.$$

Theorem 1. Let $\Psi_s = \left\{ \left([f_{\Psi_s}^-, f_{\Psi_s}^+], f_{\Psi_s} \right), \left([g_{\Psi_s}^-, g_{\Psi_s}^+], g_{\Psi_s} \right), \left([h_{\Psi_s}^-, h_{\Psi_s}^+], h_{\Psi_s} \right) \right\} (s=1,2)$ be two SCFNs and $\lambda, \lambda_1, \lambda_2 \geq 0$. Then, the following properties can be obtained:

- [1] $\Psi_1 \oplus_F \Psi_2 = \Psi_2 \oplus_F \Psi_1$.
- [2] $\Psi_1 \otimes_F \Psi_2 = \Psi_2 \otimes_F \Psi_1$.
- [3] $\lambda \bullet_F (\Psi_1 \oplus_F \Psi_2) = (\lambda \bullet_F \Psi_1) \oplus_F (\lambda \bullet_F \Psi_2)$.
- [4] $\lambda_1 \bullet_F \Psi_1 \oplus \lambda_2 \bullet_F \Psi_1 = (\lambda_1 + \lambda_2) \bullet_F \Psi_1$.
- [5] $(\Psi_1 \otimes_F \Psi_2)^{\wedge_F \lambda} = (\Psi_1)^{\wedge_F \lambda} \otimes_F (\Psi_2)^{\wedge_F \lambda}$.
- [6] $(\Psi_1)^{\wedge_F \lambda_1} \otimes_F (\Psi_1)^{\wedge_F \lambda_2} = (\Psi_1)^{\wedge_F (\lambda_1 + \lambda_2)}$.

Proof. It is trivial via Definition 11.

3.2 SCF Frank weighed average operator

The current section develops SCF Frank weighted and ordered weighted averaging operators utilizing the Frank norms.

Definition 12. Let $\Psi_s = \left\{ \left([f_{\Psi_s}^-, f_{\Psi_s}^+], f_{\Psi_s} \right), \left([g_{\Psi_s}^-, g_{\Psi_s}^+], g_{\Psi_s} \right), \left([h_{\Psi_s}^-, h_{\Psi_s}^+], h_{\Psi_s} \right) \right\} (s=1,2,\dots,n)$ be a set of SCFNs. Λ_s is the weight of SCFN Ψ_s with $\sum_{s=1}^n \Lambda_s = 1, \Lambda_s \in [0, 1]$. Then SCF Frank weighted averaging (SCFFWA) operator is depicted as:

$$SCFFWA(\Psi_1, \Psi_2, \dots, \Psi_n) = \Lambda_1 \bullet_F \Psi_1 \oplus_F \Lambda_2 \bullet_F \Psi_2 \oplus_F \dots \oplus_F \Lambda_n \bullet_F \Psi_n. \quad (10)$$

Theorem 2. Let $\Psi_s = \left\{ \left([f_s^-, f_s^+], f_s \right), \left([g_s^-, g_s^+], g_s \right), \left([h_s^-, h_s^+], h_s \right) \right\} (s=1,2,\dots,n)$ be a set of SCFNs. Then the integration outcome of SCFNs by SCFFWA operator is a SCFN and represented as

$$SCFFWA(\Psi_1, \Psi_2, \dots, \Psi_n) =$$

$$\left\{ \left[\sqrt{1 - \log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{1-(f_{\Psi_s}^-)^2} - 1 \right)^{\Lambda_s} \right)}, \sqrt{1 - \log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{1-(f_{\Psi_s}^+)^2} - 1 \right)^{\Lambda_s} \right)}, \sqrt{1 - \log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{1-(f_{\Psi_s})^2} - 1 \right)^{\Lambda_s} \right)} \right], \left[\sqrt{\log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{(g_{\Psi_s}^-)^2} - 1 \right)^{\Lambda_s} \right)}, \sqrt{\log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{(g_{\Psi_s}^+)^2} - 1 \right)^{\Lambda_s} \right)}, \sqrt{\log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{(g_{\Psi_s})^2} - 1 \right)^{\Lambda_s} \right)} \right], \left[\sqrt{\log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{(h_{\Psi_s}^-)^2} - 1 \right)^{\Lambda_s} \right)}, \sqrt{\log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{(h_{\Psi_s}^+)^2} - 1 \right)^{\Lambda_s} \right)}, \sqrt{\log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{(h_{\Psi_s})^2} - 1 \right)^{\Lambda_s} \right)} \right] \right\} \quad (11)$$

Proof. This theorem can be proved by mathematical induction. When $n = 2$, one has

$$SCFFWA(\Psi_1, \Psi_2) = \Lambda_1 \bullet_F \Psi_1 \oplus_F \Lambda_2 \bullet_F \Psi_2 =$$

$$\begin{aligned} & \left\{ \left[\sqrt{1 - \log_{\gamma} \left(1 + \frac{\left(\gamma^{1-(f_{\Psi_1}^-)^2} - 1 \right)^{\Lambda_1}}{(\gamma - 1)^{\Lambda_1 - 1}} \right)}, \sqrt{1 - \log_{\gamma} \left(1 + \frac{\left(\gamma^{1-(f_{\Psi_1}^+)^2} - 1 \right)^{\Lambda_1}}{(\gamma - 1)^{\Lambda_1 - 1}} \right)}, \sqrt{1 - \log_{\gamma} \left(1 + \frac{\left(\gamma^{1-(f_{\Psi_1})^2} - 1 \right)^{\Lambda_1}}{(\gamma - 1)^{\Lambda_1 - 1}} \right)} \right], \right. \\ & \left. \left[\sqrt{\log_{\gamma} \left(1 + \frac{\left(\gamma^{(g_{\Psi_1}^-)^2} - 1 \right)^{\Lambda_1}}{(\gamma - 1)^{\Lambda_1 - 1}} \right)}, \sqrt{\log_{\gamma} \left(1 + \frac{\left(\gamma^{(g_{\Psi_1}^+)^2} - 1 \right)^{\Lambda_1}}{(\gamma - 1)^{\Lambda_1 - 1}} \right)}, \sqrt{\log_{\gamma} \left(1 + \frac{\left(\gamma^{(g_{\Psi_1})^2} - 1 \right)^{\Lambda_1}}{(\gamma - 1)^{\Lambda_1 - 1}} \right)} \right], \right. \\ & \left. \left[\sqrt{\log_{\gamma} \left(1 + \frac{\left(\gamma^{(h_{\Psi_1}^-)^2} - 1 \right)^{\Lambda_1}}{(\gamma - 1)^{\Lambda_1 - 1}} \right)}, \sqrt{\log_{\gamma} \left(1 + \frac{\left(\gamma^{(h_{\Psi_1}^+)^2} - 1 \right)^{\Lambda_1}}{(\gamma - 1)^{\Lambda_1 - 1}} \right)}, \sqrt{\log_{\gamma} \left(1 + \frac{\left(\gamma^{(h_{\Psi_1})^2} - 1 \right)^{\Lambda_1}}{(\gamma - 1)^{\Lambda_1 - 1}} \right)} \right] \right\} \\ & \oplus_F \left\{ \left[\sqrt{1 - \log_{\gamma} \left(1 + \frac{\left(\gamma^{1-(f_{\Psi_2}^-)^2} - 1 \right)^{\Lambda_2}}{(\gamma - 1)^{\Lambda_2 - 1}} \right)}, \sqrt{1 - \log_{\gamma} \left(1 + \frac{\left(\gamma^{1-(f_{\Psi_2}^+)^2} - 1 \right)^{\Lambda_2}}{(\gamma - 1)^{\Lambda_2 - 1}} \right)}, \sqrt{1 - \log_{\gamma} \left(1 + \frac{\left(\gamma^{1-(f_{\Psi_2})^2} - 1 \right)^{\Lambda_2}}{(\gamma - 1)^{\Lambda_2 - 1}} \right)} \right], \right. \\ & \left. \left[\sqrt{\log_{\gamma} \left(1 + \frac{\left(\gamma^{(g_{\Psi_2}^-)^2} - 1 \right)^{\Lambda_2}}{(\gamma - 1)^{\Lambda_2 - 1}} \right)}, \sqrt{\log_{\gamma} \left(1 + \frac{\left(\gamma^{(g_{\Psi_2}^+)^2} - 1 \right)^{\Lambda_2}}{(\gamma - 1)^{\Lambda_2 - 1}} \right)}, \sqrt{\log_{\gamma} \left(1 + \frac{\left(\gamma^{(g_{\Psi_2})^2} - 1 \right)^{\Lambda_2}}{(\gamma - 1)^{\Lambda_2 - 1}} \right)} \right], \right. \\ & \left. \left[\sqrt{\log_{\gamma} \left(1 + \frac{\left(\gamma^{(h_{\Psi_2}^-)^2} - 1 \right)^{\Lambda_2}}{(\gamma - 1)^{\Lambda_2 - 1}} \right)}, \sqrt{\log_{\gamma} \left(1 + \frac{\left(\gamma^{(h_{\Psi_2}^+)^2} - 1 \right)^{\Lambda_2}}{(\gamma - 1)^{\Lambda_2 - 1}} \right)}, \sqrt{\log_{\gamma} \left(1 + \frac{\left(\gamma^{(h_{\Psi_2})^2} - 1 \right)^{\Lambda_2}}{(\gamma - 1)^{\Lambda_2 - 1}} \right)} \right] \right\} \\ & = \left\{ \left[\sqrt{1 - \log_{\gamma} \left(1 + \prod_{s=1}^2 \left(\gamma^{1-(f_{\Psi_s}^-)^2} - 1 \right)^{\Lambda_s} \right)}, \sqrt{1 - \log_{\gamma} \left(1 + \prod_{s=1}^2 \left(\gamma^{1-(f_{\Psi_s}^+)^2} - 1 \right)^{\Lambda_s} \right)}, \sqrt{1 - \log_{\gamma} \left(1 + \prod_{s=1}^2 \left(\gamma^{1-(f_{\Psi_s})^2} - 1 \right)^{\Lambda_s} \right)} \right], \right. \\ & \left. \left[\sqrt{\log_{\gamma} \left(1 + \prod_{s=1}^2 \left(\gamma^{(g_{\Psi_s}^-)^2} - 1 \right)^{\Lambda_s} \right)}, \sqrt{\log_{\gamma} \left(1 + \prod_{s=1}^2 \left(\gamma^{(g_{\Psi_s}^+)^2} - 1 \right)^{\Lambda_s} \right)}, \sqrt{\log_{\gamma} \left(1 + \prod_{s=1}^2 \left(\gamma^{(g_{\Psi_s})^2} - 1 \right)^{\Lambda_s} \right)} \right], \right. \\ & \left. \left[\sqrt{\log_{\gamma} \left(1 + \prod_{s=1}^2 \left(\gamma^{(h_{\Psi_s}^-)^2} - 1 \right)^{\Lambda_s} \right)}, \sqrt{\log_{\gamma} \left(1 + \prod_{s=1}^2 \left(\gamma^{(h_{\Psi_s}^+)^2} - 1 \right)^{\Lambda_s} \right)}, \sqrt{\log_{\gamma} \left(1 + \prod_{s=1}^2 \left(\gamma^{(h_{\Psi_s})^2} - 1 \right)^{\Lambda_s} \right)} \right] \right\}. \end{aligned}$$

It is supposed that Eq. (11) holds for $n = n$, namely,

$$SCFFWA(\Psi_1, \Psi_2, \dots, \Psi_n) =$$

$$\left\{ \left[\sqrt{1 - \log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{1 - (f_{\Psi_s}^-)^2} - 1 \right)^{\Lambda_s} \right)} \right], \left[\sqrt{1 - \log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{1 - (f_{\Psi_s}^+)^2} - 1 \right)^{\Lambda_s} \right)} \right], \left[\sqrt{1 - \log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{1 - (f_{\Psi_s})^2} - 1 \right)^{\Lambda_s} \right)} \right] \right\}$$

For $n = n+1$, we have

$$SCFFWA(\Psi_1, \Psi_2, \dots, \Psi_{n+1}) = \varpi_1 \bullet_F \Psi_1 \oplus_F \varpi_2 \bullet_F \Psi_2 \oplus_F \dots \oplus_F \varpi_n \bullet_F \Psi_n \oplus_F \varpi_{n+1} \bullet_F \Psi_{n+1}$$

$$\begin{aligned} & \left\{ \left[\sqrt{1 - \log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{1 - (f_{\Psi_s}^-)^2} - 1 \right)^{\Lambda_s} \right)} \right], \left[\sqrt{1 - \log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{1 - (f_{\Psi_s}^+)^2} - 1 \right)^{\Lambda_s} \right)} \right], \left[\sqrt{1 - \log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{1 - (f_{\Psi_s})^2} - 1 \right)^{\Lambda_s} \right)} \right] \right\} \\ & \oplus_F \left\{ \left[\sqrt{1 - \log_{\gamma} \left(1 + \frac{\left(\gamma^{1 - (f_{\Psi_{n+1}}^-)^2} - 1 \right)^{\Lambda_{n+1}}}{(\gamma - 1)^{\Lambda_{n+1} - 1}} \right)} \right], \left[\sqrt{1 - \log_{\gamma} \left(1 + \frac{\left(\gamma^{1 - (f_{\Psi_{n+1}}^+)^2} - 1 \right)^{\Lambda_{n+1}}}{(\gamma - 1)^{\Lambda_{n+1} - 1}} \right)} \right], \left[\sqrt{1 - \log_{\gamma} \left(1 + \frac{\left(\gamma^{1 - (f_{\Psi_{n+1}})^2} - 1 \right)^{\Lambda_{n+1}}}{(\gamma - 1)^{\Lambda_{n+1} - 1}} \right)} \right] \right\} \\ & = \left\{ \left[\sqrt{1 - \log_{\gamma} \left(1 + \prod_{s=1}^{n+1} \left(\gamma^{1 - (f_{\Psi_s}^-)^2} - 1 \right)^{\Lambda_s} \right)} \right], \left[\sqrt{1 - \log_{\gamma} \left(1 + \prod_{s=1}^{n+1} \left(\gamma^{1 - (f_{\Psi_s}^+)^2} - 1 \right)^{\Lambda_s} \right)} \right], \left[\sqrt{1 - \log_{\gamma} \left(1 + \prod_{s=1}^{n+1} \left(\gamma^{1 - (f_{\Psi_s})^2} - 1 \right)^{\Lambda_s} \right)} \right] \right\} \end{aligned}$$

Hence, Eq. (11) keeps for $n=n+1$. Thus, Eq. (11) holds for all n , which finishes the proof of Theorem 2.

Proposition 1. Let $\Psi_s = \left(\left([f_s^-, f_s^+], f_s \right), \left([g_s^-, g_s^+], g_s \right), \left([h_s^-, h_s^+], h_s \right) \right) (s=1, 2, \dots, n)$ be a set of SCFNs. Then SCFFWA operator has the following properties:

(1) (Idempotency) If all SCFNs are equal, i.e., $\Psi_s = \Psi$, $\forall s$, then one has $SCFFWA(\Psi_1, \Psi_2, \dots, \Psi_n) = \Psi$.

(2) (Boundedness) If $\Psi^- = \min(\Psi_1, \Psi_2, \dots, \Psi_n)$ and $\Psi^+ = \max\{\Psi_1, \Psi_2, \dots, \Psi_n\}$, then we have $\Psi^- \leq SCFFWA(\Psi_1, \Psi_2, \dots, \Psi_n) \leq \Psi^+$.

(3) (Monotonicity) For the set of SCFNs, $\Psi_{\tilde{\lambda}}$ and $\Psi_s (s=1, 2, \dots, n)$. If $\Psi_{\tilde{\lambda}} \leq \Psi_s$, then we have $SCFFWA(\Psi_1, \Psi_2, \dots, \Psi_n) \leq SCFFWA(\Psi_1, \Psi_2, \dots, \Psi_n)$.

Proof.

(1) Because $\Psi_s = \Psi = \left(\left([f_{\Psi}^-, f_{\Psi}^+], f_{\Psi} \right), \left([g_{\Psi}^-, g_{\Psi}^+], g_{\Psi} \right), \left([h_{\Psi}^-, h_{\Psi}^+], h_{\Psi} \right) \right)$, then

$$SCFFWA(\Psi_1, \Psi_2, \dots, \Psi_n) =$$

$$\begin{aligned} & \left(\left(\sqrt[1-\log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{1-(f_{\Psi_s}^-)^2} - 1 \right)^{\Lambda_s} \right)} \right), \left(\sqrt{\log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{(g_{\Psi_s}^-)^2} - 1 \right)^{\Lambda_s} \right)} \right), \left(\sqrt{\log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{(h_{\Psi_s}^-)^2} - 1 \right)^{\Lambda_s} \right)} \right), \right. \\ & \left. \sqrt[1-\log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{1-(f_{\Psi_s}^+)^2} - 1 \right)^{\Lambda_s} \right)} \right), \sqrt{\log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{(g_{\Psi_s}^+)^2} - 1 \right)^{\Lambda_s} \right)}, \sqrt{\log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{(h_{\Psi_s}^+)^2} - 1 \right)^{\Lambda_s} \right)} \right) \\ & = \left(\left(\sqrt[1-\log_{\gamma} \left(1 + \left(\gamma^{1-(f_{\Psi}^-)^2} - 1 \right)^{\sum_{s=1}^n \Lambda_s} \right)} \right), \left(\sqrt{\log_{\gamma} \left(1 + \left(\gamma^{(g_{\Psi}^-)^2} - 1 \right)^{\sum_{s=1}^n \Lambda_s} \right)} \right), \left(\sqrt{\log_{\gamma} \left(1 + \left(\gamma^{(h_{\Psi}^-)^2} - 1 \right)^{\sum_{s=1}^n \Lambda_s} \right)} \right), \right. \\ & \left. \sqrt[1-\log_{\gamma} \left(1 + \left(\gamma^{1-(f_{\Psi}^+)^2} - 1 \right)^{\sum_{s=1}^n \Lambda_s} \right)} \right), \sqrt{\log_{\gamma} \left(1 + \left(\gamma^{(g_{\Psi}^+)^2} - 1 \right)^{\sum_{s=1}^n \Lambda_s} \right)}, \sqrt{\log_{\gamma} \left(1 + \left(\gamma^{(h_{\Psi}^+)^2} - 1 \right)^{\sum_{s=1}^n \Lambda_s} \right)} \right) \\ & = \left(\left(\sqrt[1-\log_{\gamma} \left(\gamma^{1-(f_{\Psi}^-)^2} \right)} \right), \left(\sqrt{\log_{\gamma} \left(\gamma^{(g_{\Psi}^-)^2} \right)} \right), \left(\sqrt{\log_{\gamma} \left(\gamma^{(h_{\Psi}^-)^2} \right)} \right), \right. \\ & \left. \sqrt[1-\log_{\gamma} \left(\gamma^{1-(f_{\Psi}^+)^2} \right)} \right), \sqrt{\log_{\gamma} \left(\gamma^{(g_{\Psi}^+)^2} \right)}, \sqrt{\log_{\gamma} \left(\gamma^{(h_{\Psi}^+)^2} \right)} \right) = \left(\left([f_{\Psi}^-, f_{\Psi}^+], f_{\Psi} \right), \left([g_{\Psi}^-, g_{\Psi}^+], g_{\Psi} \right), \left([h_{\Psi}^-, h_{\Psi}^+], h_{\Psi} \right) \right) \end{aligned}$$

which finishes the Idempotency of SCFFWA operator.

(2) (Boundedness) If $\Psi^- = \min(\Psi_1, \Psi_2, \dots, \Psi_n)$ and $\Psi^+ = \max\{\Psi_1, \Psi_2, \dots, \Psi_n\}$, then we have

$$SCFFWA(\Psi_1, \Psi_2, \dots, \Psi_n) \geq SCFFWA(\Psi^-, \Psi^-, \dots, \Psi^-) = \Psi^-,$$

$$SCFFWA(\Psi_1, \Psi_2, \dots, \Psi_n) \leq SCFFWA(\Psi^+, \Psi^+, \dots, \Psi^+) = \Psi^+.$$

Hence, we have $\Psi^- \leq SCFFWA(\Psi_1, \Psi_2, \dots, \Psi_n) \leq \Psi^+$.

Definition 13. Let $\Psi_s = \left\{ \left(\left[f_{\Psi_s}^-, f_{\Psi_s}^+ \right], f_{\Psi_s} \right), \left(\left[g_{\Psi_s}^-, g_{\Psi_s}^+ \right], g_{\Psi_s} \right), \left(\left[h_{\Psi_s}^-, h_{\Psi_s}^+ \right], h_{\Psi_s} \right) \right\} (s = 1, 2, \dots, n)$ be a set of SCFNs. Λ_s is the weight of SCFN Ψ_s with $\sum_{s=1}^n \Lambda_s = 1, \Lambda_s \in [0, 1]$. Then SCF Frank ordered weighted averaging (SCFFOWA) operator is depicted as:

$$SCFFOWA(\Psi_1, \Psi_2, \dots, \Psi_n) = \varpi_1 \bullet_F \Psi_{o(1)} \oplus_F \varpi_2 \bullet_F \Psi_{o(2)} \oplus_F \dots \oplus_F \varpi_n \bullet_F \Psi_{o(n)}. \quad (12)$$

where $(o(1), o(2), \dots, o(n))$ stands for the permutation of $(s = 1, 2, \dots, n)$ with $\Psi_{o(s-1)} \geq \Psi_{o(s)}, \forall s = 2, 3, \dots, n$.

Theorem 3. Let $\Psi_s = \left\{ \left(\left[f_{\Psi_s}^-, f_{\Psi_s}^+ \right], f_{\Psi_s} \right), \left(\left[g_{\Psi_s}^-, g_{\Psi_s}^+ \right], g_{\Psi_s} \right), \left(\left[h_{\Psi_s}^-, h_{\Psi_s}^+ \right], h_{\Psi_s} \right) \right\} (s = 1, 2, \dots, n)$ be a set of SCFNs. Then the integration outcome of SCFNs by SCFFOWA operator is a SCFN and represented as:

$$SCFFOWA(\Psi_1, \Psi_2, \dots, \Psi_n) = \left(\left(\left[\sqrt{1 - \log_{\Gamma} \left(1 + \prod_{s=1}^n \left(\Gamma^{1 - (f_{o(s)}^-)^2} - 1 \right)^{\Lambda_s}} \right]}, \left[\sqrt{1 - \log_{\Gamma} \left(1 + \prod_{s=1}^n \left(\Gamma^{1 - (f_{o(s)}^+)^2} - 1 \right)^{\Lambda_s}} \right]} \right], \sqrt{1 - \log_{\Gamma} \left(1 + \prod_{s=1}^n \left(\Gamma^{1 - (f_{o(s)}}^2 - 1 \right)^{\Lambda_s}} \right)} \right), \right. \\ \left. \left(\left[\sqrt{\log_{\Gamma} \left(1 + \prod_{s=1}^n \left(\Gamma^{(g_{o(s)}^-)^2} - 1 \right)^{\Lambda_s}} \right]}, \left[\sqrt{\log_{\Gamma} \left(1 + \prod_{s=1}^n \left(\Gamma^{(g_{o(s)}^+)^2} - 1 \right)^{\Lambda_s}} \right]} \right], \sqrt{\log_{\Gamma} \left(1 + \prod_{s=1}^n \left(\Gamma^{(g_{o(s)}}^2 - 1 \right)^{\Lambda_s}} \right)} \right), \right. \\ \left. \left(\left[\sqrt{\log_{\Gamma} \left(1 + \prod_{s=1}^n \left(\Gamma^{(h_{o(s)}^-)^2} - 1 \right)^{\Lambda_s}} \right]}, \left[\sqrt{\log_{\Gamma} \left(1 + \prod_{s=1}^n \left(\Gamma^{(h_{o(s)}^+)^2} - 1 \right)^{\Lambda_s}} \right]} \right], \sqrt{\log_{\Gamma} \left(1 + \prod_{s=1}^n \left(\Gamma^{(h_{o(s)}}^2 - 1 \right)^{\Lambda_s}} \right)} \right) \right). \quad (13)$$

3.3 SCF Frank weighed geometric operator

Definition 14. Let $\Psi_s = \left\{ \left(\left[f_{\Psi_s}^-, f_{\Psi_s}^+ \right], f_{\Psi_s} \right), \left(\left[g_{\Psi_s}^-, g_{\Psi_s}^+ \right], g_{\Psi_s} \right), \left(\left[h_{\Psi_s}^-, h_{\Psi_s}^+ \right], h_{\Psi_s} \right) \right\} (s = 1, 2, \dots, n)$ be a set of SCFNs. Λ_s is the weight of SCFN Ψ_s with $\sum_{s=1}^n \Lambda_s = 1, \Lambda_s \in [0, 1]$. Then SCF Frank weighted geometric (SCFFWG) operator is depicted as:

$$SCFFWG(\Psi_1, \Psi_2, \dots, \Psi_n) = (\Psi_1)^{\wedge_F \Lambda_1} \otimes_F (\Psi_2)^{\wedge_F \Lambda_2} \dots \otimes_F (\Psi_n)^{\wedge_F \Lambda_n} \quad (14)$$

Theorem 4. Let $\Psi_s = \left\{ \left([f_s^-, f_s^+], f_s \right), \left([g_s^-, g_s^+], g_s \right), \left([h_s^-, h_s^+], h_s \right) \right\} (s=1, 2, \dots, n)$ be a set of SCFNs. Then the integration outcome of SCFNs by SCFFWG operator is a SCFN and represented as: $SCFFWG(\Psi_1, \Psi_2, \dots, \Psi_n) =$

$$\left\{ \left(\left[\sqrt{\log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{(h_{\Psi_s}^-)^2} - 1 \right)^{\Lambda_s} \right)}, \sqrt{\log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{(h_{\Psi_s}^+)^2} - 1 \right)^{\Lambda_s} \right)} \right], \sqrt{\log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{(f_{\Psi_s})^2} - 1 \right)^{\Lambda_s} \right)} \right), \left(\left[\sqrt{\log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{(g_{\Psi_s}^-)^2} - 1 \right)^{\Lambda_s} \right)}, \sqrt{\log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{(g_{\Psi_s}^+)^2} - 1 \right)^{\Lambda_s} \right)} \right], \sqrt{\log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{(f_{\Psi_s})^2} - 1 \right)^{\Lambda_s} \right)} \right), \left(\left[\sqrt{1 - \log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{1 - (g_{\Psi_s}^-)^2} - 1 \right)^{\Lambda_s} \right)}, \sqrt{1 - \log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{1 - (g_{\Psi_s}^+)^2} - 1 \right)^{\Lambda_s} \right)} \right], \sqrt{1 - \log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{1 - (g_{\Psi_s})^2} - 1 \right)^{\Lambda_s} \right)} \right) \right\} \quad (15)$$

Proposition 2. Let $\Psi_s = \left\{ \left([f_s^-, f_s^+], f_s \right), \left([g_s^-, g_s^+], g_s \right), \left([h_s^-, h_s^+], h_s \right) \right\} (s=1, 2, \dots, n)$ be a set of SCFNs. Then SCFFWG operator has the following properties:

- (1) (*Idempotency*) If all SCFNs are equal, i.e., $\Psi_s = \Psi, \forall s$, then $SCFFWG(\Psi_1, \Psi_2, \dots, \Psi_n) = \Psi$.
- (2) (*Boundedness*) If $\Psi^- = \min_{1 \leq s \leq n} \{ \Psi_s \}$ and $\Psi^+ = \max_{1 \leq s \leq n} \{ \Psi_s \}$, then $\Psi^- \leq SCFFWG(\Psi_1, \Psi_2, \dots, \Psi_n) \leq \Psi^+$.
- (3) (*Monotonicity*) For the set of SCFNs, Ψ_{λ} and $\Psi_s (s=1, 2, \dots, n)$. If $\Psi_s \leq \Psi_{\lambda}$, then we have $SCFFWG(\Psi_1, \Psi_2, \dots, \Psi_n) \leq SCFFWG(\Psi_1, \Psi_2, \dots, \Psi_n),$.

Definition 15. Let $\Psi_s = \left\{ \left([f_{\Psi_s}^-, f_{\Psi_s}^+], f_{\Psi_s} \right), \left([g_{\Psi_s}^-, g_{\Psi_s}^+], g_{\Psi_s} \right), \left([h_{\Psi_s}^-, h_{\Psi_s}^+], h_{\Psi_s} \right) \right\} (s=1, 2, \dots, n)$ be a set of SCFNs. Λ_s is the weight of SCFN Ψ_s with $\sum_{s=1}^n \Lambda_s = 1, \Lambda_s \in [0, 1]$. Then SCF Frank ordered weighted geometric (SCFFOWG) operator is depicted as:

$$SCFFOWG(\Psi_1, \Psi_2, \dots, \Psi_n) = \left(\Psi_{o(1)} \right)^{\wedge_F \Lambda_1} \otimes_F \left(\Psi_{o(2)} \right)^{\wedge_F \Lambda_2} \cdots \otimes_F \left(\Psi_{o(n)} \right)^{\wedge_F \Lambda_n} \quad (16)$$

where $(o(1), o(2), \dots, o(n))$ is the permutation of $(s=1, 2, \dots, n)$ with $\Psi_{o(s-1)} \geq \Psi_{o(s)}, \forall s=2, 3, \dots, n$.

Theorem 5. Let $\Psi_s = \left\{ \left([f_{\Psi_s}^-, f_{\Psi_s}^+], f_{\Psi_s} \right), \left([g_{\Psi_s}^-, g_{\Psi_s}^+], g_{\Psi_s} \right), \left([h_{\Psi_s}^-, h_{\Psi_s}^+], h_{\Psi_s} \right) \right\} (s=1, 2, \dots, n)$ be a set of SCFNs. Then the integration outcome of SCFNs by SCFFOWG operator is a SCFN and represented as:

$$SCFFOWG(\Psi_1, \Psi_2, \dots, \Psi_n) = \left\{ \left(\left[\sqrt{\log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{(f_{o(s)}^-)^2} - 1 \right)^{\Lambda_s} \right)}, \sqrt{\log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{(f_{o(s)}^+)^2} - 1 \right)^{\Lambda_s} \right)} \right], \sqrt{\log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{(f_{o(s)})^2} - 1 \right)^{\Lambda_s} \right)} \right), \left(\left[\sqrt{\log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{(g_{o(s)}^-)^2} - 1 \right)^{\Lambda_s} \right)}, \sqrt{\log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{(g_{o(s)}^+)^2} - 1 \right)^{\Lambda_s} \right)} \right], \sqrt{\log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{(g_{o(s)})^2} - 1 \right)^{\Lambda_s} \right)} \right), \left(\left[\sqrt{1 - \log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{1 - (h_{o(s)}^-)^2} - 1 \right)^{\Lambda_s} \right)}, \sqrt{1 - \log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{1 - (h_{o(s)}^+)^2} - 1 \right)^{\Lambda_s} \right)} \right], \sqrt{1 - \log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{1 - (h_{o(s)})^2} - 1 \right)^{\Lambda_s} \right)} \right) \right\} \quad (17)$$

4. An enhanced decision approach under SCF setting

In this section, a novel SCF-MCDM approach is proposed by integrating the LOPCOW method and an improved regret theory-based SCF-CoCoSo method. In the proposed MCDM method, the weight

information is determined using score function-based SCF-LOPCOW method, while the ranking of alternatives is ascertained by the regret theory-based SCF-CoCoSo method. To address MCDM problems under SCF information, several fundamental notions of the classical MCDM problem are preliminarily introduced. A traditional MCDM problem consists of a set of alternatives, denoted as $Q = \{Q_t | t = 1, 2, \dots, m\}$, and a set of criteria, denoted as $W = \{W_s | s = 1, 2, \dots, n\}$. It is assumed that decision experts employ SCF information to express their preferences for alternatives with respect to the evaluation criteria; accordingly, the SCF decision matrix is constructed as $E = (e_{ts})_{m \times n}$, where $e_{ts} = \{([f_{ts}^-, f_{ts}^+], f_{ts}), ([g_{ts}^-, g_{ts}^+], g_{ts}), ([h_{ts}^-, h_{ts}^+], h_{ts})\}$ is an SCFN representing the assessment of decision alternative Q_t with respect to attribute W_s . Based on the above notions, the steps of the proposed SCF-LOPCOW-RT-CoCoSo method are described as follows.

Step 1 : Obtain the SCF decision-making matrix $E = (e_{ts})_{m \times n} (t = 1, 2, \dots, m; s = 1, 2, \dots, n)$.

$$E = (e_{ts})_{m \times n} = \begin{pmatrix} \{([f_{11}^-, f_{11}^+], f_{11}), ([g_{11}^-, g_{11}^+], g_{11}), ([h_{11}^-, h_{11}^+], h_{11})\} & \cdots & \{([f_{1n}^-, f_{1n}^+], f_{1n}), ([g_{1n}^-, g_{1n}^+], g_{1n}), ([h_{1n}^-, h_{1n}^+], h_{1n})\} \\ \{([f_{21}^-, f_{21}^+], f_{21}), ([g_{21}^-, g_{21}^+], g_{21}), ([h_{21}^-, h_{21}^+], h_{21})\} & \cdots & \{([f_{2n}^-, f_{2n}^+], f_{2n}), ([g_{2n}^-, g_{2n}^+], g_{2n}), ([h_{2n}^-, h_{2n}^+], h_{2n})\} \\ \vdots & \ddots & \vdots \\ \{([f_{m1}^-, f_{m1}^+], f_{m1}), ([g_{m1}^-, g_{m1}^+], g_{m1}), ([h_{m1}^-, h_{m1}^+], h_{m1})\} & \cdots & \{([f_{mn}^-, f_{mn}^+], f_{mn}), ([g_{mn}^-, g_{mn}^+], g_{mn}), ([h_{mn}^-, h_{mn}^+], h_{mn})\} \end{pmatrix} \quad (18)$$

Step 2: Since the criteria in an MCDM problem can be classified into benefit-type and cost-type criteria, it is necessary to convert the cost-type criteria into benefit-type criteria and subsequently derive the normalized SCF decision matrix. Accordingly, the SCF decision matrix $E = (e_{ts})_{m \times n}$ is transformed into the normalized SCF decision matrix (NSCFDM) $\tilde{E} = (\tilde{e}_{ts})_{m \times n}$ by the following formulation:

$$\tilde{e}_{st} = \begin{cases} e_{ts} = \{([f_{ts}^-, f_{ts}^+], f_{ts}), ([g_{ts}^-, g_{ts}^+], g_{ts}), ([h_{ts}^-, h_{ts}^+], h_{ts})\}, \text{ for benefit criteria} \\ (\tilde{e}_{st})^c = \{([h_{ts}^-, h_{ts}^+], h_{ts}), ([\varepsilon_{ts}^-, \varepsilon_{ts}^+], \varepsilon_{ts}), ([f_{ts}^-, f_{ts}^+], f_{ts})\}, \text{ for cost criteria} \end{cases} \quad (19)$$

Step 3. Determine the criterion weights.

LOPCOW is an emerging objective weighting method proposed in 2022. This method assigns criterion weights according to logarithmic percentage changes, thereby determining the importance of each evaluation indicator by capturing the variation information embedded in the decision matrix. In this study, the LOPCOW method is improved by employing the SCF score function to objectively determine the importance of the criteria. Let $\Lambda = (\Lambda_1, \Lambda_2, \dots, \Lambda_n)^T$ be the weight vector of the assessment criteria, satisfying the constraint $\sum_{s=1}^n \Lambda_s = 1, \Lambda_s \in [0, 1]$. The procedure of the SCF-LOPCOW algorithm is described as follows.

Step 1: Compute the SCF score matrix $R = (r_{ts})_{m \times n}$ of the group decision matrix.

$$r_{ts} = SF(\tilde{e}_{ts}) = \frac{1}{2} \left(1 + \frac{1}{9} \left(\left(f_{ts}^- + f_{ts}^+ + f_{ts} \right)^2 + \left(g_{ts}^- + g_{ts}^+ + g_{ts} \right)^2 - \left(h_{ts}^- + h_{ts}^+ + h_{ts} \right)^2 \right) \right) \quad (20)$$

Step 2: Calculate the standard deviation β_j and the root mean square value T_j of the normalized assessments with respect to the evaluation criteria W_s , respectively.

$$\beta_s = \sqrt{\frac{\sum_{t=1}^m \left(r_{ts} - \frac{1}{m} \sum_{t=1}^m r_{ts} \right)^2}{m}}, (s = 1(1)n), \quad (21)$$

$$T_s = \sqrt{\frac{1}{m} \sum_{t=1}^m (r_{ts})^2}, (s = 1(1)n). \quad (22)$$

Step 3: Calculate the percentage value for each criterion using the following formula:

$$Y_s = \left| \ln \left(\frac{T_s}{\beta_s} \right) \right| \cdot 100, s = 1(1)n. \quad (23)$$

Step 4: Determine the final weights of the evaluation criteria:

$$\Lambda_s = \frac{Y_s}{\sum_{s=1}^n Y_s}. \quad (24)$$

Step 5: Compute the sum of the weighted comparability sequence Δ_t and power-weighted comparability sequence ∇_t by Eq. (25) and Eq. (26).

$$SCF\Delta(Q_i) = SCFFWA(\tilde{e}_{i1}, \tilde{e}_{i2}, \dots, \tilde{e}_{im}) = \left(\left[\sqrt{1 - \log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{1-(f_{ts}^-)^2} - 1 \right)^{\Lambda_s} \right)}, \sqrt{1 - \log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{1-(f_{ts}^+)^2} - 1 \right)^{\Lambda_s} \right)}, \sqrt{1 - \log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{1-(f_{ts})^2} - 1 \right)^{\Lambda_s} \right)} \right], \left[\sqrt{\log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{(g_{ts}^-)^2} - 1 \right)^{\Lambda_s} \right)}, \sqrt{\log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{(g_{ts}^+)^2} - 1 \right)^{\Lambda_s} \right)}, \sqrt{\log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{(g_{ts})^2} - 1 \right)^{\Lambda_s} \right)} \right], \left[\sqrt{\log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{(\bar{h}_{ts}^-)^2} - 1 \right)^{\Lambda_s} \right)}, \sqrt{\log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{(\bar{h}_{ts}^+)^2} - 1 \right)^{\Lambda_s} \right)}, \sqrt{\log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{(\bar{h}_{ts})^2} - 1 \right)^{\Lambda_s} \right)} \right] \right) \quad (25)$$

$$SCF\nabla(Q_i) = SCFFWG(\tilde{e}_{i1}, \tilde{e}_{i2}, \dots, \tilde{e}_{im}) = \left(\left[\sqrt{\log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{(f_{ts}^-)^2} - 1 \right)^{\Lambda_s} \right)}, \sqrt{\log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{(f_{ts}^+)^2} - 1 \right)^{\Lambda_s} \right)}, \sqrt{\log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{(f_{ts})^2} - 1 \right)^{\Lambda_s} \right)} \right], \left[\sqrt{\log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{(g_{ts}^-)^2} - 1 \right)^{\Lambda_s} \right)}, \sqrt{\log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{(g_{ts}^+)^2} - 1 \right)^{\Lambda_s} \right)}, \sqrt{\log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{(g_{ts})^2} - 1 \right)^{\Lambda_s} \right)} \right], \left[\sqrt{1 - \log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{1-(\bar{h}_{ts}^-)^2} - 1 \right)^{\Lambda_s} \right)}, \sqrt{1 - \log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{1-(\bar{h}_{ts}^+)^2} - 1 \right)^{\Lambda_s} \right)}, \sqrt{1 - \log_{\gamma} \left(1 + \prod_{s=1}^n \left(\gamma^{1-(\bar{h}_{ts})^2} - 1 \right)^{\Lambda_s} \right)} \right] \right) \quad (26)$$

Where Λ_s is the criteria weight with $\sum_{s=1}^n \Lambda_s = 1$, $\Lambda_s \in [0, 1]$.

Step 6: Calculate the utility values of each alternative. According to the risk aversion coefficient δ of the expert, we determine the utility value of the alternative through Eq. (27) and Eq. (28). The ideal point of the alternative can be determined by Eq. (29) and Eq. (30).

$$SCF\Delta U_t = \left(SF \left(SCF\Delta(Q_t) \right) \right)^\delta, t = 1, 2, \dots, m \quad (27)$$

$$SCF\nabla U_t = \left(SF \left(SCF\nabla(Q_t) \right) \right)^\delta, t = 1, 2, \dots, m \quad (28)$$

$$SCF\Delta U^* = \max_{1 \leq t \leq m} \left(SF \left(\left(SCF\Delta(Q_t) \right)^\delta \right) \right) \quad (29)$$

$$SCF\nabla U^* = \max_{1 \leq t \leq m} \left(SF \left(\left(SCF\nabla(Q_t) \right)^\delta \right) \right) \quad (30)$$

Step 7: Based on the regret aversion coefficient ε and ideal point, the regret value of every alternative can be determined by Eq. (31) and Eq. (32).

$$SCF\Delta \mathfrak{R}_t = 1 - \exp \left(-\varepsilon \left(SCF\Delta U_t - SCF\Delta U^* \right) \right) \quad (31)$$

$$SCF\nabla \mathfrak{R}_t = 1 - \exp \left(-\varepsilon \left(SCF\nabla U_t - SCF\nabla U^* \right) \right) \quad (32)$$

Step 8. The overall utility value of every alternative can be determined by Eq. (33) and Eq. (34).

$$SCF\Delta U_t = SCF\Delta U_t + SCF\Delta \mathfrak{R}_t \quad (33)$$

$$SCF\nabla U_t = SCF\nabla U_t + SCF\nabla \mathfrak{R}_t \quad (34)$$

Step 9. Calculate the final score value for the alternative. The arithmetic mean of each alternative is computed by Eq. (35), The sum of the relative scores between the alternative and the optimal solution alternative is computed by Eq. (36), and the equilibrium compromise values for each solution is computed by Eq. (37).

$$M_t^1 = \frac{SCF\Delta U_t + SCF\nabla U_t}{\sum_{t=1}^m (SCF\Delta U_t + SCF\nabla U_{ti})}, t = 1, 2, \dots, m \quad (35)$$

$$M_t^2 = \frac{SCF\Delta U_t}{\min_{1 \leq t \leq m} (SCF\Delta U_t)} + \frac{SCF\nabla U_t}{\min_{1 \leq t \leq m} (SCF\nabla U_t)}, t = 1, 2, \dots, m \quad (36)$$

$$M_t^3 = \frac{\vartheta SCF\Delta U_t + (1 - \vartheta) SCF\nabla U_t}{\vartheta \max_{1 \leq t \leq m} (SCF\Delta U_t) + (1 - \vartheta) \max_{1 \leq t \leq m} (SCF\nabla U_t)}, \vartheta \in [0, 1]. \quad (37)$$

Step 10. The final score for each alternative is computed by Eq. (38). The importance of alternatives is ranked by the score, where the higher scores have higher importance of alternative.

$$M_t = \sqrt[3]{M_t^1 M_t^2 M_t^3} + \frac{1}{3} (M_t^1 + M_t^2 + M_t^3) \quad (38)$$

5. Case study

In this section, we use the case of cardiovascular disease diagnosis in patients to validate the validity and performance of the proposed SCF-LOPCOW-RT-CoCoSo method.

Cardiovascular disease is one of the deadliest diseases worldwide and has attracted much attention from scholars and doctors. According to statistics from the World Health Organization, approximately 17.9 million people die from cardiovascular disease worldwide each year. This figure accounts for 32% of the total global deaths and making cardiovascular disease the leading cause of death worldwide. With the outstanding development of precision medicine, the diagnosis and prevention of cardiovascular diseases have also received attention. Early precision diagnosis plays an important positive role in preventing and treating cardiovascular diseases in patients. The diagnosis of cardiovascular disease is essentially a multi-attribute decision-making problem, with decision information characterized by multi-source, high uncertainty and ambiguity. In clinical practice process, doctors need to comprehensively judge whether a patient has cardiovascular disease based on multiple indicators, the most common of which are troponin, myoglobin, blood lipids, liver function, kidney function, electrolytes, myocardial enzymes and serum cholesterol. However, due to the dynamic changes in patients' health status and inherent limitations of detection methods, the data of various indicators often exhibit fluctuating characterize and doctors' interpretation of the results also has linguistic ambiguity. Therefore, SCF sets are highly suitable for characterizing uncertainty in cardiovascular disease diagnosis information. A medical expert was invited to diagnose five patients (Q_1, Q_2, Q_3, Q_4, Q_5) who may have cardiovascular disease, and selected four main indicators (W_1, W_2, W_3, W_4), including Blood routine examination, myocardial enzymes, resting blood pressure and serum cholesterol for diagnosing cardiovascular disease as diagnostic attributes, and the weight information of the diagnostic attributes was completely unknown. Based on these four diagnostic attributes, experts used SCF numbers (Table 1) to provide diagnostic information for 5 patients under the diagnostic attributes and then conducted decision analysis using the proposed SCF-LOPCOW-RT CoCoSo method.

Table 1

Linguistic terms and their corresponding SCF numbers

Extremely high importance	EHI	$\langle ([0.8, 0.9], 0.85), ([0.0, 0.1], 0.05), ([0.0, 0.1], 0.05) \rangle$
high importance	HI	$\langle ([0.6, 0.7], 0.65), ([0.2, 0.3], 0.25), ([0.2, 0.3], 0.25) \rangle$
Slightly high importance	SHI	$\langle ([0.5, 0.6], 0.55), ([0.3, 0.4], 0.35), ([0.3, 0.4], 0.35) \rangle$
Medium importance	MI	$\langle ([0.4, 0.5], 0.45), ([0.4, 0.5], 0.45), ([0.4, 0.5], 0.45) \rangle$
Slightly low importance	SLI	$\langle ([0.3, 0.4], 0.35), ([0.3, 0.4], 0.35), ([0.5, 0.6], 0.55) \rangle$
low importance	LI	$\langle ([0.2, 0.3], 0.25), ([0.2, 0.3], 0.25), ([0.2, 0.7], 0.65) \rangle$
Very low importance	VLI	$\langle ([0.1, 0.2], 0.15), ([0.1, 0.2], 0.15), ([0.1, 0.8], 0.75) \rangle$
Extremely low importance	ELI	$\langle ([0.0, 0.1], 0.05), ([0.0, 0.1], 0.05), ([0.0, 0.9], 0.85) \rangle$

5.1 Decision analysis process

Step 1: Obtain the SCF decision-making matrix $E = (e_{ts})_{m \times n}$ based on experts' cognition and displayed in Table 2.

Step 2: Since Blood routine examination (W_1), myocardial enzymes (W_2), Resting blood pressure (W_3) and serum cholesterol (W_4) are all benefit-type criteria. Hence the normalized SCF decision matrix is same with the original decision matrix.

Step 3. Determine the criterion weights.

Step 3.1: Determine the SCF score matrix $R = (r_{ts})_{m \times n}$ of the normalized decision matrix by Eq. (20), Table 3.

Table 2
SCF decision-making matrix

	Blood routine examination (W ₁)	myocardial enzymes (W ₂)	Resting blood pressure (W ₃)	Serum cholesterol (W ₄)
Q_1	< ([0.7,0.8],0.75), ([0.1,0.2],0.15), ([0.1,0.2],0.15) >	< ([0.5,0.6],0.55), ([0.3,0.4],0.35), ([0.3,0.4],0.35) >	< ([0.6,0.7],0.65), ([0.2,0.3],0.25), ([0.2,0.3],0.25) >	< ([0.6,0.7],0.65), ([0.2,0.3],0.25), ([0.2,0.3],0.25) >
Q_2	< ([0.7,0.8],0.75), ([0.1,0.2],0.15), ([0.1,0.2],0.15) >	< ([0.5,0.6],0.55), ([0.3,0.4],0.35), ([0.3,0.4],0.35) >	< ([0.2,0.3],0.25), ([0.2,0.3],0.25), ([0.2,0.7],0.65) >	< ([0.5,0.6],0.55), ([0.3,0.4],0.35), ([0.3,0.4],0.35) >
Q_3	< ([0.6,0.7],0.65), ([0.2,0.3],0.25), ([0.2,0.3],0.25) >	< ([0.7,0.8],0.75), ([0.1,0.2],0.15), ([0.1,0.2],0.15) >	< ([0.7,0.8],0.75), ([0.1,0.2],0.15), ([0.1,0.2],0.15) >	< ([0.2,0.3],0.25), ([0.2,0.7],0.65) >
Q_4	< ([0.7,0.8],0.75), ([0.1,0.2],0.15), ([0.1,0.2],0.15) >	< ([0.4,0.5],0.45), ([0.4,0.5],0.45), ([0.4,0.5],0.45) >	< ([0.4,0.5],0.45), ([0.4,0.5],0.45), ([0.4,0.5],0.45) >	< ([0.7,0.8],0.75), ([0.1,0.2],0.15), ([0.1,0.2],0.15) >
Q_5	< ([0.6,0.7],0.65), ([0.2,0.3],0.25), ([0.2,0.3],0.25) >	< ([0.2,0.3],0.25), ([0.2,0.3],0.25), ([0.2,0.7],0.65) >	< ([0.5,0.6],0.55), ([0.3,0.4],0.35), ([0.3,0.4],0.35) >	< ([0.4,0.5],0.45), ([0.4,0.5],0.45), ([0.4,0.5],0.45) >

Table 3
SCF score matrix $R = (r_{is})_{m \times n}$ of the normalized decision matrix

	Blood routine examination (W ₁)	myocardial enzymes (W ₂)	Resting blood pressure (W ₃)	Serum cholesterol (W ₄)
Q_1	0.7813	0.6513	0.7113	0.7113
Q_2	0.7813	0.6513	0.4290	0.6513
Q_3	0.7113	0.7813	0.7813	0.4290
Q_4	0.7813	0.6013	0.6013	0.7813
Q_5	0.7113	0.4290	0.6513	0.6013

Step 3.2: The standard deviation β_s and the root mean square value T_s of the normalized decision matrix are computed by Eq. (21) and Eq. (22). The results are listed as $\beta_1 = 0.0343$, $\beta_2 = 0.1138$, $\beta_3 = 0.1192$, $\beta_4 = 0.1192$; $T_1 = 0.7540$, $T_2 = 0.6331$, $T_3 = 0.6459$, $T_4 = 0.6459$.

Step 3.3: The percentage value for each criterion is computed by Eq. (23) and listed as $Y_1 = 309.0495$, $Y_2 = 171.6324$, $Y_3 = 168.9927$, $Y_4 = 168.9927$.

Step 3.4: The final weights of the evaluation criteria are figured out by Eq. (24) and listed as $\Lambda_1 = 0.3775$, $\Lambda_2 = 0.2096$, $\Lambda_3 = 0.2064$, $\Lambda_4 = 0.2064$,

Step 4: The weighted comparability value Δ_i and power-weighted comparability value ∇_i are computed by Eq. (25) and Eq. (26). The results are listed as:

$$\begin{aligned}
 SCF\Delta(Q_1) &= \{([0.7890, 0.8516], 0.8208), ([0.4111, 0.5242], 0.4718), ([0.4111, 0.5242], 0.4718)\}, \\
 SCF\Delta(Q_2) &= \{([0.7336, 0.8043], 0.7694), ([0.4294, 0.5406], 0.4891), ([0.4294, 0.5942], 0.5442)\}, \\
 SCF\Delta(Q_3) &= \{([0.7640, 0.8317], 0.7983), ([0.3880, 0.5041], 0.4503), ([0.3880, 0.5552], 0.5022)\}, \\
 SCF\Delta(Q_4) &= \{([0.7711, 0.8380], 0.8049), ([0.4273, 0.5457], 0.4917), ([0.4273, 0.5457], 0.4917)\}, \\
 SCF\Delta(Q_5) &= \{([0.6843, 0.7566], 0.7212), ([0.5024, 0.5960], 0.5514), ([0.5024, 0.6539], 0.6121)\}.
 \end{aligned}$$

$$\begin{aligned}
SCF\nabla(Q_1) &= \{([0.7833, 0.8451], 0.8148), ([0.4111, 0.5242], 0.4718), ([0.4306, 0.5347], 0.4854)\}, \\
SCF\Delta(Q_2) &= \{([0.6929, 0.7676], 0.7317), ([0.4294, 0.5406], 0.4891), ([0.4549, 0.6409], 0.5980)\}, \\
SCF\Delta(Q_3) &= \{([0.7227, 0.7604], 0.7983), ([0.3880, 0.5041], 0.4503), ([0.3991, 0.6062], 0.5604)\}, \\
SCF\Delta(Q_4) &= \{([0.7485, 0.7823], 0.8049), ([0.4273, 0.5457], 0.4917), ([0.4841, 0.5802], 0.5342)\}, \\
SCF\Delta(Q_5) &= \{([0.6550, 0.7322], 0.6950), ([0.5024, 0.5960], 0.5514), ([0.5148, 0.6811], 0.6415)\}. \\
SF(SCF\Delta(Q_1)) &= 0.8366, \quad SF(SCF\Delta(Q_2)) = 0.7775, \quad SF(SCF\Delta(Q_3)) = 0.8025, \\
SF(SCF\Delta(Q_4)) &= 0.8238, \quad SF(SCF\Delta(Q_5)) = 0.7372. \quad SF(SCF\Delta(Q_1)) = 0.8366, \\
SF(SCF\Delta(Q_2)) &= 0.7775, \quad SF(SCF\Delta(Q_3)) = 0.8025, \quad SF(SCF\Delta(Q_4)) = 0.8238, \\
SF(SCF\Delta(Q_5)) &= 0.7372.
\end{aligned}$$

Step 5: The initial utility values of each alternative are computed by Eq. (27) and Eq. (28), respectively. Here, parameter is set as $\delta = 0.88$. The outcomes mentioned are listed as:

$$\begin{aligned}
SCF\Delta U_1 &= 0.8547, SCF\Delta U_2 = 0.8013, SCF\Delta U_3 = 0.8239, SCF\Delta U_4 = 0.8431, \\
SCF\Delta U_5 &= 0.7647. SCF\nabla U_1 = 0.8440, SCF\nabla U_2 = 0.7543, SCF\nabla U_3 = 0.7785, \\
SCF\nabla U_4 &= 0.8061, SCF\nabla U_5 = 0.7347. \text{ The ideal point of the alternative can be determined by} \\
\text{Eq. (29) and Eq. (30), severally, which is listed as } SCF\Delta U^* &= 0.8547, SCF\nabla U^* = 0.8440.
\end{aligned}$$

Step 6: Based on the regret aversion coefficient ε ($\varepsilon = 0.5$) and ideal point, the regret value of every alternative can be determined by Eq. (31) and Eq. (32). The outcomes are listed as:

$$\begin{aligned}
SCF\Delta \mathfrak{R}_1 &= 0.0000, SCF\Delta \mathfrak{R}_2 = -0.0270, SCF\Delta \mathfrak{R}_3 = -0.0155, SCF\Delta \mathfrak{R}_4 = -0.0058, \\
SCF\Delta \mathfrak{R}_5 &= -0.0460; SCF\nabla \mathfrak{R}_1 = 0.0000, SCF\nabla \mathfrak{R}_2 = -0.0459, SCF\nabla \mathfrak{R}_3 = -0.0333, \\
SCF\nabla \mathfrak{R}_4 &= -0.0191, SCF\nabla \mathfrak{R}_5 = -0.0562.
\end{aligned}$$

Step 7: The overall utility value of every alternative can be determined by Eq. (33) and Eq. (34).

$$\begin{aligned}
SCF\Delta U_1 &= 0.8547, SCF\Delta U_2 = 0.7743, SCF\Delta U_3 = 0.8085, SCF\Delta U_4 = 0.8374, \\
SCF\Delta U_5 &= 0.7186, SCF\nabla U_1 = 0.8440, SCF\nabla U_2 = 0.7085, SCF\nabla U_3 = 0.7452, \\
SCF\nabla U_4 &= 0.7870, SCF\nabla U_5 = 0.6786.
\end{aligned}$$

Step 8-9: The final score value for the alternative is computed by Eq. (35), The sum of the relative scores between the alternative and the optimal solution alternative is computed by Eq. (36), and the equilibrium compromise values for each solution is computed by Eq. (37). The final score for each alternative is computed by Eq. (38). The outcomes mentioned are collected in Table 4. The diagnosis outcome of five patients is $Q_1 \succ Q_4 \succ Q_3 \succ Q_2 \succ Q_5$. Therefore, according to the diagnosis and analysis of medical experts, patient Q_1 are most likely to have cardiovascular disease, while patient Q_5 is least likely to have cardiovascular disease, which means that patient Q_1 urgently needs treatment.

5.2 Parameter discussions and analysis

In the proposed SCF-LOPCOW-RT-CoCoSo decision method, there are two parameters, which maybe influence the final ranking outcomes. Hence, we discuss the impact of parameter values Υ and ϑ on decision results.

Case-1: Discussion of parameter Υ in the proposed SCF operators. We validate the trend of parameter Υ on the final ranking of patients through setting the values of parameter Υ from 3 to 20. The final utility grades and related ranking are listed in Table 4 and Figure 1. From the results, we

can find the diagnosis outcome of five patients is $Q_1 \succ Q_4 \succ Q_3 \succ Q_2 \succ Q_5$, which did not change with the variation of parameter Υ (Table 5). Hence, the proposed SCF-LOPCOW-RT-CoCoSo decision method is stability and robustness.

Table 4
Final score of alternatives from three aggregation strategies

	M_t^1	M_t^2	M_t^3	M_t	Ranking
Q_1	0.2190	2.4331	1.0000	2.0281	1
Q_2	0.1912	2.1216	0.8729	1.7693	4
Q_3	0.2003	2.2231	0.9146	1.8539	3
Q_4	0.2094	2.3251	0.9563	1.9387	2
Q_5	0.1801	2.0000	0.8225	1.6676	5

Table 5
Decision outcomes by different values of parameter Υ

Υ	Q_1	Q_2	Q_3	Q_4	Q_5	Ranking
3	2.0281	1.7693	1.8539	1.9387	1.6676	$Q_1 \succ Q_4 \succ Q_3 \succ Q_2 \succ Q_5$
4	2.0275	1.7698	1.8545	1.9396	1.6677	$Q_1 \succ Q_4 \succ Q_3 \succ Q_2 \succ Q_5$
5	2.0270	1.7701	1.8549	1.9404	1.6678	$Q_1 \succ Q_4 \succ Q_3 \succ Q_2 \succ Q_5$
6	2.0267	1.7704	1.8551	1.9409	1.6679	$Q_1 \succ Q_4 \succ Q_3 \succ Q_2 \succ Q_5$
7	2.0264	1.7706	1.8553	1.9414	1.6680	$Q_1 \succ Q_4 \succ Q_3 \succ Q_2 \succ Q_5$
8	2.0262	1.7707	1.8554	1.9418	1.6680	$Q_1 \succ Q_4 \succ Q_3 \succ Q_2 \succ Q_5$
9	2.0260	1.7708	1.8555	1.9421	1.6681	$Q_1 \succ Q_4 \succ Q_3 \succ Q_2 \succ Q_5$
10	2.0258	1.7709	1.8555	1.9424	1.6681	$Q_1 \succ Q_4 \succ Q_3 \succ Q_2 \succ Q_5$
20	2.0249	1.7713	1.8556	1.9440	1.6684	$Q_1 \succ Q_4 \succ Q_3 \succ Q_2 \succ Q_5$

Case-2: Discussion of parameter ϑ in the proposed SCF-CoCoSo method. The parameter reflects the decision preference of experts in determining the equilibrium compromise solution. We also validate the trend of parameter ϑ on the final ranking of patients through setting the values of parameter Υ from 0.1 to 1. The final utility grades and related ranking are listed in Figure 2. From the results, we can find the diagnosis outcome of five patients is $Q_1 \succ Q_4 \succ Q_3 \succ Q_2 \succ Q_5$, which implies that the proposed SCF-LOPCOW-RT-CoCoSo decision method is stability and robustness.

5.3 Comparative analysis

This part presents a comparison analysis among the proposed SCF-LOPCOW-RT-CoCoSo decision method and the existing SCF methods including SCFWA operator-based method [9], SCFWG operator-based method [9] and SCF-WASPAS approach [29]. The comparison outcomes of the mentioned approaches are shown in Figure 3. It can be found that the final ranking attained by the mentioned approaches are all $Q_1 \succ Q_4 \succ Q_3 \succ Q_2 \succ Q_5$, which can imply that the consistency and effectiveness of the proposed approach. Although the best options are all same, the proposed SCF-LOPCOW-RT-CoCoSo decision method takes into consideration the flexibility of decision process and psychological behavior of experts. The decision outcome has stronger reliability and generality than other SCF methods.

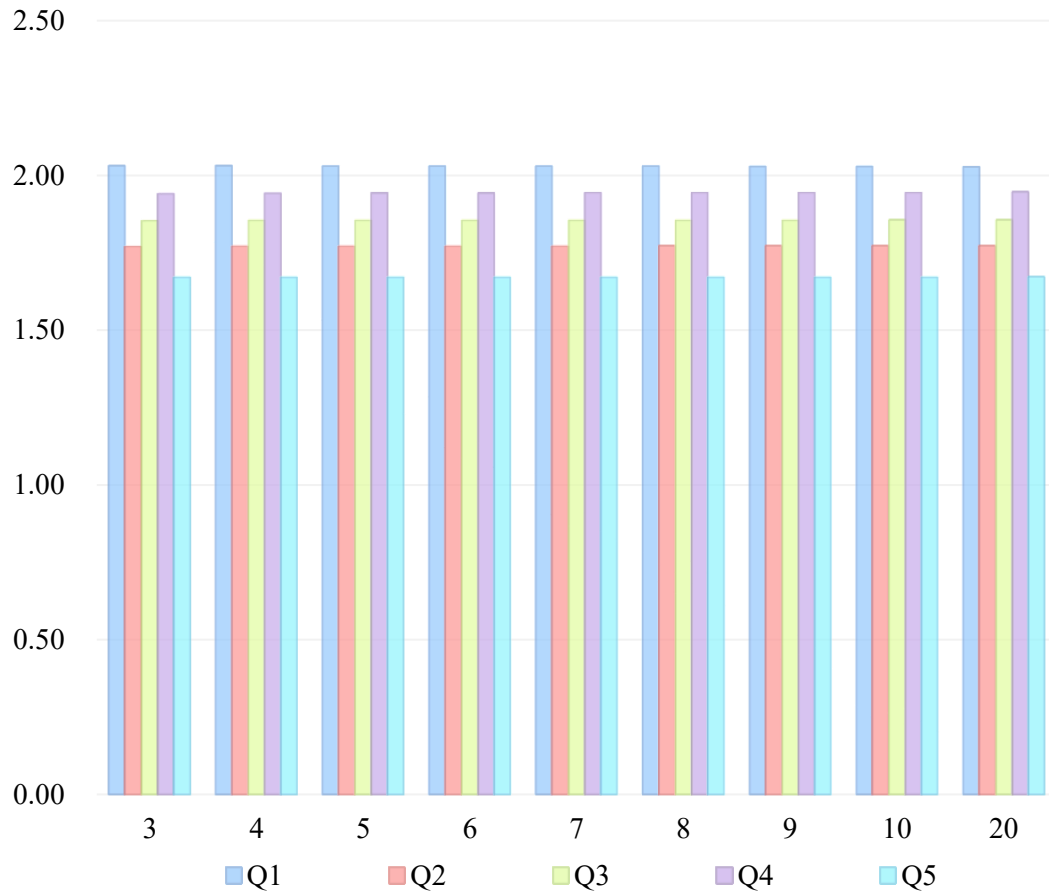


Fig. 1. The change of final ranking of patients by diverse values of parameter Y

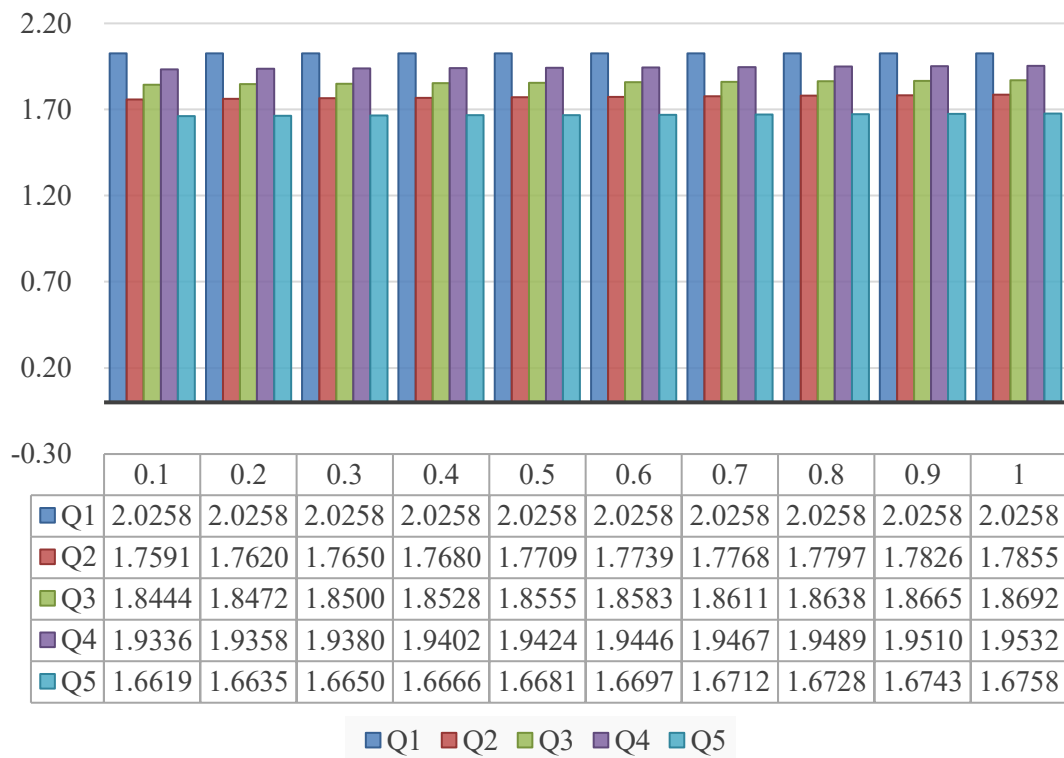


Fig. 2. The change of final ranking of patients by diverse values of parameter J

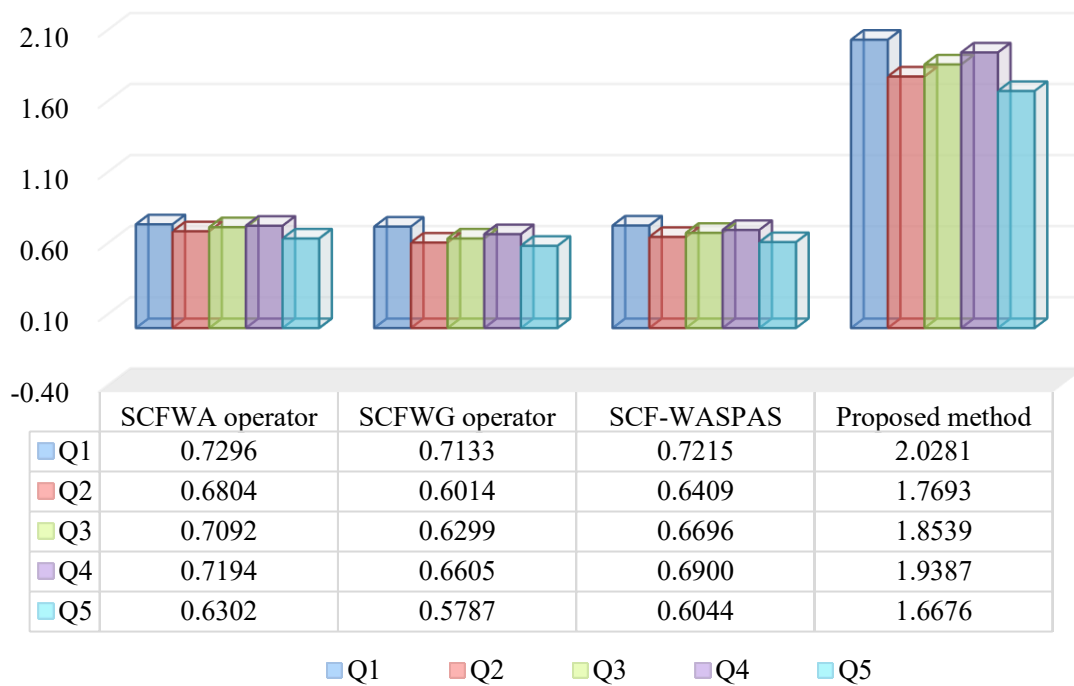


Fig. 3. Comparison outcomes of the proposed method and prior SCF decision approaches

6. Conclusion

This article proposes an SCF-LOPCOW-RT-CoCoSo decision method based on Frank aggregation operator and regret theory to address the problems of insufficient consideration of expert psychological behavior in existing research on SCF sets. The main conclusions are as follows. The SCF operations based on Frank norms and four SCF aggregation operators defined in this study can effectively integrate uncertain preference information provided by multiple experts, which further enhances the flexibility of the SCF information fusion process. We extend the LOPCOW method to the SCF environment and combines it with the SCF score function to determine criterion weights, which improves the objectivity and adaptability of weight determination. We construct an improved CoCoSo model that considers the psychological behavior of experts, so that the decision process not only focuses on the aggregation of evaluation information, but also on the psychological reactions of experts in uncertain environments. The robustness and effectiveness of the proposed SCF-LOPCOW-RT-CoCoSo method in uncertain environments were verified through the application of cardiovascular disease diagnosis cases, sensitivity and comparative analysis. Research has shown that this method can provide new ideas and methodological support for complex decision problems.

Although the method proposed in this article has certain advantages in SCF decision-making, it can still be further expanded. Future research can apply this framework to more practical decision-making scenarios, such as risk assessment, emergency decision-making, and supplier selection, and consider dynamic decision environments, group decision-making models based on social network trust relationships, and more psychological behavior theories to further enhance the applicability and explanatory power of the method.

Funding

This research is supported by Ningxia Medical University Campus level Scientific Research Project (XT2025053) and Ningxia Natural Science Foundation Project (2026AAC030265).

Data availability

Data will be made available on request.

References

- [1] Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, 8(3), 338–353. [https://doi.org/10.1016/S0019-9958\(65\)90241-X](https://doi.org/10.1016/S0019-9958(65)90241-X)
- [2] Atanassov, K. T. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 20(1), 87–96. [https://doi.org/10.1016/S0165-0114\(86\)80034-3](https://doi.org/10.1016/S0165-0114(86)80034-3)
- [3] Yager, R. R. (2014). Pythagorean membership grades in multicriteria decision making. *IEEE Transactions on Fuzzy Systems*, 22(4), 958–965. <https://doi.org/10.1109/TFUZZ.2013.2278989>
- [4] Yager, R. R. (2017). Generalized orthopair fuzzy sets. *IEEE Transactions on Fuzzy Systems*, 25(5), 1222–1230. <https://doi.org/10.1109/TFUZZ.2016.2604005>
- [5] Cuong, B. C. (2014). Picture fuzzy sets. *Journal of Computer Science and Cybernetics*, 30(4), 409–420. <https://doi.org/10.15625/1813-9663/30/4/5032>
- [6] Rong, Y., Yu, L., Niu, W., Liu, Y., Senapati, T., & Mishra, A. R. (2022). MARCOS approach based upon cubic Fermatean fuzzy set and its application in evaluation and selecting cold chain logistics distribution center. *Engineering Applications of Artificial Intelligence*, 116, 105401. <https://doi.org/10.1016/j.engappai.2022.105401>
- [7] Rong, Y., Yu, L., Liu, Y., Simic, V., & Pamucar, D. (2024). A pharmaceutical cold-chain logistics service quality model using a q-rung orthopair fuzzy framework with distance measure. *Engineering Applications of Artificial Intelligence*, 136, 109019. <https://doi.org/10.1016/j.engappai.2024.109019>
- [8] Rong, Y., Yu, L., Liu, Y., Peng, X., & Garg, H. (2024). An integrated group decision-making framework for assessing S3PRLPs based on MULTIMOORA-WASPAS with q-rung orthopair fuzzy information. *Artificial Intelligence Review*, 57(6). <https://doi.org/10.1007/s10462-024-10782-7>
- [9] Ayaz, T., Al-Shomrani, M. M., Abdullah, S., & Hussain, A. (2020). Evaluation of enterprise production based on spherical cubic Hamacher aggregation operators. *Mathematics*, 8(10), 1761. <https://doi.org/10.3390/math8101761>
- [10] Ali, G., Nabeel, M., & Farooq, A. (2024). Extended ELECTRE method for multi-criteria group decision-making with spherical cubic fuzzy sets. *Knowledge and Information Systems*, 66(10), 6269–6306. <https://doi.org/10.1007/s10115-024-02132-4>
- [11] Mohammed, M. T., Shubber, M. S., Qahtan, S., Alsatta, H. A., Mourad, N., Zaidan, A. A., & Zaidan, B. B. (2025). Determining the superiority of a robust cloud fault tolerance mechanism using a spherical cubic fuzzy set-based decision approach. *Engineering Applications of Artificial Intelligence*, 148, 110402. <https://doi.org/10.1016/j.engappai.2025.110402>
- [12] Opricovic, S., & Tzeng, G. H. (2004). Compromise solution by MCDM methods: A comparative analysis of VIKOR and TOPSIS. *European Journal of Operational Research*, 156(2), 445–455. [https://doi.org/10.1016/S0377-2217\(03\)00020-1](https://doi.org/10.1016/S0377-2217(03)00020-1)
- [13] Mardani, A., Nilashi, M., Zakuan, N., Loganathan, N., Soheilrad, S., Saman, M. Z. M., & Ibrahim, O. (2017). A systematic review and meta-analysis of SWARA and WASPAS methods: Theory and applications with recent fuzzy developments. *Applied Soft Computing*, 57, 265–292. <https://doi.org/10.1016/j.asoc.2017.03.045>
- [14] Zavadskas, E. K., Turskis, Z., & Vilutiene, T. (2010). Multiple criteria analysis of foundation instalment alternatives by applying Additive Ratio Assessment (ARAS) method. *Archives of Civil and Mechanical Engineering*, 10(3), 123–141. [https://doi.org/10.1016/S1644-9665\(12\)60141-1](https://doi.org/10.1016/S1644-9665(12)60141-1)
- [15] Guo, F., Gao, J., Liu, H., & He, P. (2021). Locations appraisal framework for floating photovoltaic power plants based on relative-entropy measure and improved hesitant fuzzy linguistic DEMATEL-PROMETHEE method. *Ocean & Coastal Management*, 215, 105948. <https://doi.org/10.1016/j.ocecoaman.2021.105948>
- [16] Yazdani, M., Zarate, P., Zavadskas, E. K., & Turskis, Z. (2019). A combined compromise solution (CoCoSo) method for multi-criteria decision-making problems. *Management Decision*, 57(9), 2501–2519. <https://doi.org/10.1108/MD-05-2017-0458>
- [17] Ecer, F., & Pamucar, D. (2020). Sustainable supplier selection: A novel integrated fuzzy best worst method (F-BWM) and fuzzy CoCoSo with Bonferroni (CoCoSo'B) multi-criteria model. *Journal of Cleaner Production*, 266, 121981. <https://doi.org/10.1016/j.jclepro.2020.121981>
- [18] Peng, X., & Luo, Z. (2021). Decision-making model for China's stock market bubble warning: The CoCoSo with picture fuzzy information. *Artificial Intelligence Review*, 54(8), 5675–5697. <https://doi.org/10.1007/s10462-021-09954-6>
- [19] Pamucar, D., & Gorcun, O. F. (2022). Evaluation of the European container ports using a new hybrid fuzzy LBWA-CoCoSo'B techniques. *Expert Systems with Applications*, 203, 117463. <https://doi.org/10.1016/j.eswa.2022.117463>

- [20] Wang, H., Mahmood, T., & Ullah, K. (2023). Improved CoCoSo method based on Frank softmax aggregation operators for T-spherical fuzzy multiple attribute group decision-making. *International Journal of Fuzzy Systems*, 25(3), 1275–1310. <https://doi.org/10.1007/s40815-022-01442-5>
- [21] Rong, Y., & Yu, L. (2023). Decision support system for prioritization of offshore wind farm site by utilizing picture fuzzy combined compromise solution group decision method. *Entropy*, 25(7), 1081. <https://doi.org/10.3390/e25071081>
- [22] Yu, L., Kang, X., & Qiu, R. (2025). Unlocking corporate green governance capabilities in China: A supply chain-oriented evaluation based on an improved FBWM-CoCoSo method. *International Journal of Industrial Engineering Computations*, 16(4), 897–920. <https://doi.org/10.5267/j.ijiec.2025.8.010>
- [23] Khan, A., Barukab, O., Jun, Y. B., Khan, S. A., Ahmad, U., & Rushdi, A. M. A. (2025). An extended CoCoSo method under (p-q) rung orthopair fuzzy environment for multi-criteria decision-making applications. *Scientific Reports*, 15(1). <https://doi.org/10.1038/s41598-025-06020-x>
- [24] Zhang, H., & Wei, G. (2023). Location selection of electric vehicles charging stations by using the spherical fuzzy CPT-CoCoSo and D-CRITIC method. *Computational & Applied Mathematics*, 42(1). <https://doi.org/10.1007/s40314-022-02183-9>
- [25] Fan, J., Duan, Y., & Wu, M. (2025). A new approach to failure mode and effect analysis based on regret theory and group satisfaction index in basin-type insulators. *Engineering Applications of Artificial Intelligence*, 152, 110729. <https://doi.org/10.1016/j.engappai.2025.110729>
- [26] Jun, Y. B. (2017). A novel extension of cubic sets and its applications in BCK/BCI-algebras. *Annals of Fuzzy Mathematics and Informatics*, 14(5), 475–486. <https://doi.org/10.30948/afmi.2017.14.5.475>
- [27] Mahmood, T., Ullah, K., Khan, Q., & Jan, N. (2019). An approach toward decision-making and medical diagnosis problems using the concept of spherical fuzzy sets. *Neural Computing & Applications*, 31(11), 7041–7053. <https://doi.org/10.1007/s00521-018-3521-2>
- [28] Qin, J., Liu, X., & Pedrycz, W. (2016). Frank aggregation operators and their application to hesitant fuzzy multiple attribute decision making. *Applied Soft Computing*, 41, 428–452. <https://doi.org/10.1016/j.asoc.2015.12.030>
- [29] Li, D., Wan, G., & Rong, Y. (2026). An enhanced spherical cubic fuzzy WASPAS method and its application for the assessment of service quality of crowdsourcing logistics platform. *Spectrum of Decision Making and Applications*, 3(1), 100–123. <https://doi.org/10.31181/sdmap31202641>